Course Syllabus
MATH 204: Algebraic Foundations for the Middle Level Teacher

Catalog Description with Prerequisites
This course is designed for middle level (4-8) teacher candidates. It contains a concrete study of algebraic structures encountered in the middle level school mathematics curriculum. Content includes sequential patterns, and examples and properties of rings and integral domains such as the integers, integers mod n, polynomials, and matrices.
Prerequisite: passing score on BST, and C or better in MATH 104 or department permission

This course is designed to meet the requirements for the algebra content area required by students from the School of Education seeking certification as a Middle Level teacher. The course will initially be offered once every three semesters rotating with MATH 205 and MATH 230. The schedule will be adjusted as the demand is determined.

Algebra is the language of mathematics. As such, this course will build upon the elementary foundation developed in Math 104, deepening future middle level teachers' understandings of algebraic structures, representations and properties, while extending their knowledge to include introductory topics from number theory and abstract algebra. The course will focus on the development of deeper understandings of the material, examination of multiple solution strategies, and the proper verbal and written communication of covered topics. In this way, the course will prepare future middle level teachers to understand the material and better teach the content in their future profession. This aspect, as well as the level and organization of topics presented, will distinguish it from the other courses being offered in the Mathematics Department. There is no comparable course from other departments at Millersville University. No courses will be removed from the catalog.

When appropriate, instructional techniques and means of assessment will extend beyond regular exams and homework sets to include collaborative learning, hands-on activities, and real-world applications as promoted by the National Council of Teachers of Mathematics (NCTM) Principles and Standards and the Pennsylvania Department of Education Academic Standards for Mathematics. Lab activities and projects will explore course topics and provide the opportunity for students to develop new and deeper understandings of algebraic ideas and structures. These lab activities may include such topics as: clock arithmetic activities connected to modular arithmetic; the use of algebra tiles to explore ideas of factoring; and the use of graphing calculators when working with matrices.

Primary Course Objectives

Course Objectives: Upon completion of the course, students will be able to:

Objective	**Forms of Assessment**
1. Identify, represent, and construct arithmetic and algebraic patterns and sequences.	Exams, homework sets, and lab exercises
2. Identify, derive and apply arithmetic and algebraic properties of the ring of integers to problems and proofs.	Exams, homework sets, and lab exercises
3. Explain and apply the Division and Euclidean Algorithms.	Exams, homework sets, and lab exercises
4. Identify, derive and apply arithmetic and algebraic properties of the ring of integers modulo n to problems and proofs.	Exams, homework sets, and lab exercises
5. Identify, derive and apply arithmetic and algebraic properties of the ring of polynomials to problems and proofs.	Exams, homework sets, and lab exercises
6. Identify, derive and apply arithmetic and algebraic properties of the ring of matrices to problems, including solving systems of linear equations.	Exams, homework sets, and lab exercises

7. Develop and investigate mathematical conjectures.	Homework sets, projects, and lab exercises
8. Develop and evaluate algebraic arguments and proofs.	Exams, homework sets, and lab exercises
9. Properly communicate algebraic concepts orally and in written form using proper terminology and algebraic notation.	Exams, homework sets, projects, and lab exercises
10. Identify and develop connections between algebraic topics and other disciplines	Homework sets, projects, and lab exercises

In addition to the above course objectives, MATH 204 will help teacher candidates at the Middle Level to fulfill university, state, and national standards as indicated below.

I. In alignment with **Millersville University's Professional Education Unit's Conceptual Framework**, this course will serve to satisfy key indicators for Proficiency Area I: Content Knowledge. Candidates will display knowledge of the mathematical content and apply the important principles and concepts delineated in professional, state, and institutional standards.
1Specifically candidates will:
- Demonstrate competency in their chosen content area.
- Engage in inquiry in their content area that develops their ability to extend student understanding beyond surface information.
- Understand, explain, and apply knowledge of the contextual issues (e.g., political, social, cultural, ethnicity, disability, and gender) that influence their content area.
- Recognize various theories and points of view within their field.
- Develop curricula in a variety of instructional formats reflective of state, national, and local standards.

II. **PDE Standards** that teacher candidates will be better prepared to help their own students to achieve after completing this course include the following:
- PDE 2.4.8 D: Construct, use and explain algorithmic procedures for computing and estimating with whole numbers, fractions, decimals and integers.
- PDE 2.5.8 A: Invent, select, use and justify the appropriate methods, materials and strategies to solve problems.
- PDE 2.5.8 A: Invent, select, use and justify the appropriate methods, materials and strategies to solve problems.
- PDE 2.5.8 C: Justify strategies and defend approaches used and conclusions reached.
- PDE 2.8.8 A: Apply simple algebraic patterns to basic number theory and to spatial relations
- PDE 2.8.8 B: Discover, describe and generalize patterns, including linear, exponential and simple quadratic relationships.
- PDE 2.8.8 C: Create and interpret expressions, equations or inequalities that model problem situations.

III. Content Knowledge Standard for mathematics teachers from **National Council of Teachers of Mathematics for the National Council for the Accreditation of Teacher Educators** (2003).
Standard 10: Knowledge of Different Perspectives on Algebra: Candidates emphasize relationships among quantities including functions, ways of representing mathematical relationships, and the analysis of change.
Indicators
- 10.1 Analyze patterns, relations, and functions of one and two variables.
- 10.2 Apply fundamental ideas of linear algebra.
- 10.3 Apply the major concepts of abstract algebra to justify algebraic operations and formally analyze algebraic structures.

- 10.4 Use mathematical models to represent and understand quantitative relationships.
- 10.5 Use technological tools to explore algebraic ideas and representations of information and in solving problems.
- 10.6 Demonstrate knowledge of the historical development of algebra including contributions from diverse cultures.

Comprehensive Outline of Course Content

1. Patterns and Sequences *(2 weeks)*
    a. Representing Patterns and Sequences
    b. Arithmetic Sequences
    c. Geometric Sequences
    d. Mathematical Induction
    e. Fundamental Counting Principles
    f. The Binomial Theorem
    g. The Fibonacci Sequence
    h. Rings, Integral Domains and Fields
2. Arithmetic and Algebra of the Integers *(1 week)*
    a. Multiples and Divisors
    b. Least Common Multiples
    c. Greatest Common Divisors
    d. The Fundamental Theorem of Arithmetic
3. The Division Algorithm and the Euclidean Algorithm *(2 weeks)*
    a. The Division Algorithm
    b. The Euclidean Algorithm
    c. Place Value
    d. Prime Numbers and Properties
4. Arithmetic and Algebra of the Integers Modulo n *(3 weeks)*
    a. Divisibility Tests
    b. Clock Addition
    c. Modular Arithmetic
    d. Comparison of Properties of $\mathbf{Z}$ and $\mathbf{Z}_n$
    e. Multiplicative Inverses in $\mathbf{Z}_n$
    f. Elementary Applications of Modular Arithmetic
    g. Fermat's Little Theorem
    h. Wilson's Theorem
5. Arithmetic and Algebra of Polynomials *(3 weeks)*
    a. Properties of the Polynomial Ring F[x]
    b. Polynomial Arithmetic and the Division Algorithm
    c. Divisibility in F[x] and Factoring
    d. The Fundamental Theorem of Algebra
    e. Irreducible Polynomials and Unique Factorization
    f. Polynomial Functions, Roots, and Reducibility
    g. Comparison of Properties of $\mathbf{Z}$ and F[x]
6. Arithmetic and Algebra of Matrices *(3 weeks)*
    a. Systems of Linear Equations
    b. Rational and Irrational Numbers
    c. Substitution and Addition/Subtraction Methods
    d. Matrix Arithmetic and Matrix Algebra
    e. Multiplicative Inverses with Matrices
    f. Coding with Matrices

g. Comparison of Properties of Integers and Matrices

Criteria for Evaluating Student Performance

Evaluation of student learning will be accomplished by the following:

- Two or three in-class examinations and a comprehensive final exam.
- In-class lab activities involving collaboration and hands-on manipulatives.
- Homework sets to be completed outside of class.
- Individual and/or group projects connecting course content to middle school curriculum and other disciplines.

Required Materials

Textbook:

Papick, I.J. (2007). *Algebra Connections: Mathematics for Middle School Teachers.* Upper Saddle River, New Jersey: Pearson Education, Inc.

Supplemental material will be necessary for material regarding the arithmetic and algebra of polynomials.

Graphing Calculator: A TI-84 or comparable graphing calculator.

General Education Credit?

This course will satisfy G2 credit for MATH.

Resources

1. Greenes, Carole (editor); *Navigating through Algebra in Prekindergarten-Grade2*; NCTM; 2001
2. Cuevas, Gilbert J. (editor); *Navigating through Algebra in Grades 2-5*; NCTM; 2001
3. Friel, Susan (editor); *Navigating through Algebra in Grades 6-8*; NCTM; 2001
4. Lott, JohnnynW. (editor); *Navigating through Algebra in Grades 9-12*; NCTM; 2001
5. Benson, John, et. al.; *Gateways to Algebra and Geometry An Integrated Approach*; McDougal, Littell & Company; Evanston, Illinois; 1993
6. Saxon, John H. Jr.; *Algebra ½, I, II, An Incremental Development*; Grassdale Publishers, Inc. 1984
7. McConnell, John W., et. al.; Algebra; The University of Chicago School Mathematics Project; Scott, Foresman and Company; Glenview, Illinois; 1990.
8. Billstein, Rick and Jim Williamson; *Middle Grades Math Thematics, Books 2 and 3*; The STEM Project; McDougal Littell;
Evanston, Illinois; 1999