MATH 204 – Algebraic Foundations for the Middle Level Teacher – SYLLABUS

Department of Mathematics
Millersville University

Description

This course is designed for middle level (4-8) teacher candidates. It contains a concrete study of algebraic structures encountered in the middle level school mathematics curriculum. Content includes sequential patterns, and examples and properties of rings and integral domains such as the integers, integers mod n, polynomials, and matrices. (3 credits)

This course may be taken for general education credit (G2).

Prerequisites

MATH 104 (C or better) or department permission.

Course Objectives

By the conclusion of this course the successful student will be able to:

- Identify, represent, and construct arithmetic and algebraic patterns and sequences.
- Identify, derive and apply arithmetic and algebraic properties of the ring of integers to problems and proofs.
- Explain and apply the Division and Euclidean Algorithms.
- Identify, derive and apply arithmetic and algebraic properties of the ring of integers modulo n to problems and proofs.
- Identify, derive and apply arithmetic and algebraic properties of the ring of polynomials to problems and proofs.
- Identify, derive and apply arithmetic and algebraic properties of the ring of matrices to problems, including solving systems of linear equations.
- Develop and investigate mathematical conjectures.
- Develop and evaluate algebraic arguments and proofs.
- Properly communicate algebraic concepts orally and in written form using proper terminology and algebraic notation.
- Identify and develop connections between algebraic topics and other disciplines.

Assessment

Assessment of student achievement of the course objectives will vary from one instructor to another. Typical assessment will be made through work in class, homework, and examinations.

Use of Technology

Technology use will vary by professor.

Topics

1. Patterns and Sequences
    a. Representing Patterns and Sequences
    b. Arithmetic Sequences
    c. Geometric Sequences
    d. Mathematical Induction
    e. Fundamental Counting Principles
    f. The Binomial Theorem
    g. The Fibonacci Sequence
    h. Rings, Integral Domains and Fields
2. Arithmetic and Algebra of the Integers
    a. Multiples and Divisors
    b. Least Common Multiples
    c. Greatest Common Divisors
    d. The Fundamental Theorem of Arithmetic
3. The Division Algorithm and the Euclidean Algorithm
    a. The Division Algorithm
    b. The Euclidean Algorithm
    c. Place Value
    d. Prime Numbers and Properties
4. Arithmetic and Algebra of the Integers Modulo n
    a. Divisibility Tests
    b. Clock Addition
    c. Modular Arithmetic
    d. Comparison of Properties of $\mathbf{Z}$ and $\mathbf{Z}_n$
    e. Multiplicative Inverses in $\mathbf{Z}_n$
    f. Elementary Applications of Modular Arithmetic
    g. Fermat's Little Theorem
    h. Wilson's Theorem
5. Arithmetic and Algebra of Polynomials
    a. Properties of the Polynomial Ring F[x]
    b. Polynomial Arithmetic and the Division Algorithm
    c. Divisibility in F[x] and Factoring
    d. The Fundamental Theorem of Algebra
    e. Irreducible Polynomials and Unique Factorization
    f. Polynomial Functions, Roots, and Reducibility
    g. Comparison of Properties of $\mathbf{Z}$ and F[x]
6. Arithmetic and Algebra of Matrices
    a. Systems of Linear Equations
    b. Rational and Irrational Numbers
    c. Substitution and Addition/Subtraction Methods
    d. Matrix Arithmetic and Matrix Algebra

e. Multiplicative Inverses with Matrices
f. Coding with Matrices
g. Comparison of Properties of Integers and Matrices