MATH 353 – SURVEY OF GEOMETRY – SYLLABUS

Department of Mathematics
Millersville University

Description

Various examples of axiom systems, axiomatic development of neutral geometry followed by Euclidean and hyperbolic geometry. Models for Euclidean and hyperbolic Geometry. Emphasis on proving geometric theorems, both orally and in writing. (3 credits)

This course cannot be taken for general education credit.

Prerequisites

A C- or better in both Math 310 and Math 322 or permission of instructor.

Course Objectives

Students will learn the theory and techniques of calculus and its applications. By the conclusion of this course the successful student will be able to:

- Appreciate the historical significance of Euclid's axiomatic treatment of geometry.
- Demonstrate an understanding of axiomatic systems through various examples.
- Have a clear understanding of the set of axioms in neutral, Euclidean, and hyperbolic geometry.
- Understand the models for Euclidean and hyperbolic geometry.
- Write proofs in the context of neutral, Euclidean, and hyperbolic geometry.

Assessment

Assessment of student achievement of the course objectives will vary from one instructor to another. Typical assessment will be made through work in class, homework, and examinations.

Use of Technology

Mathematical technology (such as calculators or software) are not typically needed for this class, but may be required at the discretion of the instructor.

Topics

1. Euclid’s Elements
    a. The historical significance of Euclid’s Elements
    b. A look at Book I of Elements
    c. A critique of Euclid’s Elements
2. Axiomatic Systems and Incidence Geometry
    a. The structure of an axiomatic system
    b. An example: incidence geometry
    c. The parallel postulates in incidence geometry
    d. Some theorems from incidence geometry
3. Axioms of Plane Geometry
    a. The undefined terms and two fundamental axioms
    b. Distance and the Ruler Postulate
    c. The Plane Separation Postulate
    d. Angle measure and the Protractor Postulate
    e. The Crossbar Theorem and the Linear Pair Theorem
    f. The Side-Angle-Side Postulate
    g. The Parallel Postulates and models
4. Neutral Geometry
    a. The Exterior Angle Theorem and existence of perpendiculars
    b. Triangle congruence conditions
    c. Three inequalities for triangles
    d. The Alternate Interior Angles Theorem
    e. The Saccheri-Legendre Theorem
    f. Quadrilaterals
    g. Statements equivalent to the Euclidean Parallel Postulate
    h. Rectangles and defect
    i. The Universal Hyperbolic Theorem
5. Euclidean Geometry
    a. Basic theorems in Euclidean geometry
    b. The Parallel Projection Theorem
    c. Similar triangles
    d. The Pythagorean Theorem
6. Hyperbolic Geometry
    a. Basic theorems in hyperbolic geometry
    b. Common perpendiculars
    c. The angle of parallelism
    d. Limiting parallel rays
    e. The classification of parallels

7. Circles
    a. Circles and lines in neutral geometry
    b. Circles and triangles in neural geometry
    c. Circles in Euclidean geometry
8. Models
    a. The Cartesian model for Euclidean geometry
    b. The Poincare disk model of hyperbolic geometry