

BIOLOGY 100
SOLUTIONS TO PROBLEMS

LABORATORY SKILLS AND HYPOTHESIS TESTING

1. Mary bought six marbles. A list of their weights (masses) in grams is: 6.07, 7.12, 6.23, 6.89, 6.67, and 7.30. By using volume displacement, Mary determined each marble's volume in ml: 1.2, 1.2, 1.1, 1.3, 1.4, and 1.5. a. What is the average (mean) weight of the marbles? b. What is the average (mean) volume of the marbles? c. What is the average density (g/ml) of the marbles? Be sure that all your answers have been rounded correctly to the appropriate precision.

a. To obtain the mean weight, add up the weights and divide by six: $(6.07 + 7.12 + 6.23 + 6.89 + 6.67 + 7.30)/6 = 40.28/6 = 6.7133333 = 6.71$ grams.

b. To obtain the mean volume, add up the volumes and divide by six: $(1.2 + 1.2 + 1.1 + 1.3 + 1.4 + 1.5)/6 = 7.7/6 = 1.283333 = 1.3$ ml.

c. To obtain the average density, divide the mean weight by the mean volume: $6.71 \text{ g}/1.3 \text{ ml} = 5.1615384 = 5.2$ g/ml

2. How many heads and how many tails would you need in 100 coin flips to obtain a χ^2 value greater than 3.84, a result that would indicate that the coin was not normal?

Some thoughtful trial and error could lead you to 60 heads : 40 tails (or 40 heads and 60 tails). Perform the chi-square analysis on these numbers.

	Observed (O)	Expected (E)	$(O-E)^2/E$
Heads	60	50	$(60-50)^2/50 = 2$
Tails	40	50	$(40-50)^2/50 = 2$
Total	100	100	4

Here the chi-square is 4, and the critical chi-square with a degrees of freedom of 1 (2 categories - 1 = 1) is 3.84. Therefore, a 60 to 40 ratio is not supportive of a fair coin because the calculated value exceeds the critical value. If the tosses had been 59 to 41, then the calculated chi square value would have been 3.24 (less than 3.84), and therefore the data would have supported a fair coin.

So, any set of 100 tosses in which heads (or tails) is greater or equal to 60 or less than or equal to 40 would suggest an abnormality in the coin or the tosses themselves.

3. A six-sided cube with 2 red sides, 2 green sides, 1 yellow side, and 1 blue side is rolled 600 times. a. How many times would you expect to have the cube land yellow side up? b. Red side up? (Hint: The expected numbers for each color should represent the fraction of the number of sides that have that color.)

a. The cube has 6 sides in which 1 is yellow; therefore, one-sixth of the rolls should result in yellow.

$1/6 \times 600$ rolls = 100 rolls would be expected to turn up yellow.

b. The cube has 6 sides in which 2 are red; therefore, two-sixths (one-third) of the rolls should result in red.

$1/3 \times 600$ rolls = 200 rolls would be expected to turn up red.

4. Suppose the six-sided cube as described above was rolled 360 times. The results of the rolls were as follows: 111 reds, 128 greens, 52 yellows, and 69 blues. Perform a chi square analysis on these data, hypothesizing that the cube is fairly weighted, and the expected results should be appropriate for the number of sides with each of the four colors. Write down your conclusions from the chi square analysis. Do the data support your hypothesis? Explain.

See above (question 3) for how to determine expected values for each color. Set up a table for calculating chi square values for the results above.

	Observed (O)	Expected (E)	$(O-E)^2/E$
Red	111	120	$(111-120)^2/120 = 0.68$
Green	128	120	$(128-120)^2/120 = 0.53$
Yellow	52	60	$(52-60)^2/60 = 1.07$
Blue	69	60	$(69-60)^2/60 = 1.35$
Total	360	360	$0.68+0.53+1.07+1.35 = 3.63$

Therefore, the calculated chi-square value is 3.63. The degrees of freedom (DF) is 3 because four categories (colors) minus one is three. At DF = 3, the critical chi-square is 7.81 (see critical value tables in the Laboratory Skills Exercises in this manual). Since the calculated value of 3.63 is less than the critical value of 7.81, the observed data do support the hypothesis that the cube is fairly weighted.

5. Using cardboard, scotch tape, and colored construction paper, Jamie built a six-sided cube with 3 yellow sides, 1 purple side, 1 green side, and 1 orange side. To see whether the cube was weighted fairly, she rolled it 420 times. The results of the rolls were as follows: 238 yellows, 66 purples, 52 greens, and 64 oranges. Perform a chi square analysis on these results, hypothesizing that the cube was fairly weighted, and the expected results should be appropriate for the number of sides with each of the four colors. Write down your conclusions from the chi square analysis. Did Jamie construct a fairly weighted cube? Explain how the chi square test helped you answer this question to a statistical certainty.

If we hypothesize that the cube is fairly weighted, then we would expect the number of rolls for each color to be in the same proportion as the colors in the cube. So, yellow should appear $1/2$ the time (3 sides are yellow out of 6) or in 210 out of 420 rolls; purple, green, and orange should each appear $1/6$ of the time since there is only 1 side of each of these colors. That would mean we would expect to see purple or green or orange in $1/6 \times 420 = 70$ rolls of the 420 rolls. Now a chi-square can be performed:

	Observed (O)	Expected (E)	$(O-E)^2/E$
Yellow	238	210	$(238-210)^2/210 = 3.73$
Purple	66	70	$(66-70)^2/70 = 0.23$
Green	52	70	$(52-70)^2/70 = 4.63$
Orange	64	70	$(64-70)^2/70 = 0.51$
Total	420	420	$3.73+0.23+4.63+0.51 = 9.10$

Therefore, the calculated chi-square value is 9.10. The degrees of freedom (DF) is 3 because four categories (colors) minus one is three. At $DF = 3$, the critical chi-square is 7.81 (see critical value tables in the Laboratory Skills Exercises in this manual). Since the calculated value of 9.10 is greater than the critical value of 7.81, the observed data do NOT support the hypothesis that the cube is fairly weighted. This means that there is a good possibility that the cube is not fairly weighted or that something went wrong with the data collection.