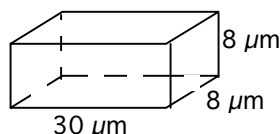**BIOLOGY 100**
SOLUTIONS TO PROBLEMS

CELL STRUCTURE

1. A cell is 8 μm in width and depth, and 30 μm in length. What is the surface area of this cell? What is the volume of this cell? What is the surface area to volume ratio of this cell?

Consider the geometry of this cell:

8 μm
8 μm
30 μm

Surface Area = surface area of 2 ends + surface area of 4 sides
= 2 x (8 μm x 8 μm) + 4 x (8 μm x 30 μm) = 1,088 μm²

Volume = base x height x depth
= 8 μm x 8 μm x 30 μm = 1,920 μm³

Surface Area to Volume ratio = Surface Area ÷ Volume
= 1,088 μm² ÷ 1,920 μm³ = 0.57 μm⁻¹

Note: $\frac{1}{\mu m}$ is the same as μm⁻¹

2. If a cubical cell maintained its shape while it grew to ten times its initial size, by what percent would the surface to volume ratio change?

The original cell is a cube and has sides with the dimension of 1 arbitrary unit (units are not shown below).
Surface Area of original cell = total surface area of the six panels that form the cube
= 6 x (1 x 1) = 6
Volume of original cell = side³ = 1³ = 1 x 1 x 1 = 1
Surface Area : Volume of original cell = 6 ÷ 1 = 6

The enlarged cell is also a cube but has sides ten times larger.
Surface Area of enlarged cell = 6 x (10 x 10) = 600
Volume of enlarged cell = 10³ = 1,000
Surface Area : Volume of enlarged cell = 600 ÷ 1,000 = 0.6

Comparison of Surface Area : Volumes = $\frac{SA : V \text{ enlarged}}{SA : V \text{ original}}$ = $\frac{0.6}{6}$ = 0.1

Hence, the SA : V for the enlarged cell is only 0.1 (10%) of the original.

3. If spherical cell A had a diameter of one unit and possessed 200 active transport proteins to pump nutrients into the cell and cell B had a diameter of 4 units, how many more active transport proteins would cell B need to provide the same nourishment to its cytoplasm? (Note: the formula for the surface area of a sphere is: $4\pi r^2$, and the formula for the volume of a sphere is: $4/3\pi r^3$, where r is the radius of the sphere and π is 3.1416.)

The transport proteins are supplying the cell with nutrients. The 'amount' of cell cytoplasm being supplied is reflected by the cell's volume.

Cell A   active transport proteins = 200
            Volume = $4/3\pi r^3$ (remember, radius = diameter ÷ 2)
                   = 4/3 x 3.14 x (0.5 units)^3
                   = 0.52 unit^3
Relationship of transport proteins to cell volume = 200 proteins/0.52 unit^3 = 385 proteins/unit^3
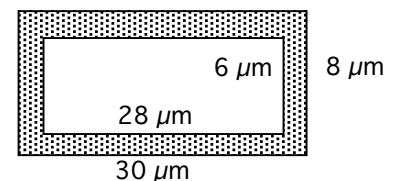
Cell B   Volume = 4/3 x 3.14 x (2 units)^3
                   = 33.49 unit^3

Transport proteins needed to supply cytoplasm of Cell B
                               = 33.49 unit^3 x 385 proteins/unit^3 = 12,894 proteins.

4. If a plant cell is 8 μm in width and depth and has a length of 30 μm, what is the surface to volume ratio for this cell? If the same cell has a large central vacuole, so that the cytoplasm (not including the vacuole) extends inward 1 μm from the plasma membrane of the cell, what is the surface to cytoplasmic volume ratio? What does this tell you about the function of the plant vacuole?

Surface Area to Volume ratio = 1,088 μm^2 ÷ 1,920 μm^3 = 0.57 μm^{-1} (same as question #1)

Think of the second part of the problem as a smaller box (vacuole) nested inside of a larger one (entire cell). The volume of the cytoplasm (shaded area) will be the volume of the larger box minus the volume of the smaller one (third dimension is not shown):

Volume of larger box = 8 *μm* x 8 *μm* x 30 *μm* = 1,920 *μm*^{3}
Volume of smaller box = 6 *μm* x 6 *μm* x 28 *μm* = 1,008 *μm*^{3}
Volume of cytoplasm = 1,920 *μm*^{3} – 1,008 *μm*^{3} = 912 *μm*^{3}

Surface Area to cytoplasmic volume ratio = 1,088 *μm*^{2} ÷ 912 *μm*^{3} = 1.19 *μm*$^{-1}$

The cell surface area to cytoplasmic volume ratio is over twice that (209%) of the surface area to volume ratio of the whole cell. One function of the vacuole is to increase the ratio of the surface area to cytoplasmic volume.

5. A cell and its nucleus both have a spherical shape; the nucleus is 1 μm in diameter and the cell is 15 μm in diameter. What is the cellular/nuclear volume ratio for this cell? If the cell were to triple its diameter, how would the cellular/nuclear volume ratio change? (Note: the formula for the surface area of a sphere is: $4\pi r^2$, and the formula for the volume of a sphere is: $4/3\pi r^3$, where r is the radius of the sphere and π is 3.1416.)

Nuclear volume = 4/3 x π x (0.5 μm)³ = 4/3 x π x 0.125 μm³
Cell volume   = 4/3 x π x (7.5 μm)³ = 4/3 x π x 421.875 μm³

Cell : Nuclear volume ratio = <u>4/3 x π x 421.875 μm³</u> = 3,375
                           4/3 x π x 0.125 μm³

In other words, the nucleus must control a cytoplasmic area that is 3,375 times its own size.

If the cell triples in diameter (assume the nucleus stays the same size since the cell will still have the same amount of genetic material):

3x-cell volume (r = 7.5 μm x 3) = 4/3 x π x (22.5 μm)³ = 4/3 x π x 11,390.625 μm³

3x-cell : nuclear volume ratio = <u>4/3 x π x 11,390.625 μm³</u> = 91,125
                              4/3 x π x 0.125 μm³

In the 3x cell, the nucleus must now control an area of cytoplasm 91,125 times its own size. Note that the cell increased its size (diameter) by only 3 times, but the cell : nuclear volume ratio became 27 times greater, i.e., the area controlled by the nucleus increased 27 fold.