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Name: Key

MATH 130 – ELEMENTS OF STATISTICS
EXAM II – VERSION 1/2

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October 29, 2009

SHOW ALL WORK NEATLY and clearly indicate your answers. Show all your working for every problem. A correct answer with no work shown (except on problems which are trivial) will receive no credit. If you are not sure if you have written enough, please ask. There are 9 problems on 5 pages. The time allowed is 75 minutes and the exam is worth 100 points. *Write your name on every page in the space provided.* You may use a calculator for this test, but not a cell-phone or laptop. You may also use a 3x5 inch formula card, and you may use the tables in the back of your textbook to calculate the probabilities for the binomial and normal distributions.

Good luck!!

1. (8 points) Indicate for each of the following variables whether the variable being described is *discrete* or *continuous*.
   a) The number of traffic accidents in Lancaster today. a) d
   b) The amount of time between accidents in Lancaster today. b) c
   c) The weight of each vehicle involved in an accident today. c) c
   d) The number of passengers in each vehicle involved in an accident today. d) d

2. (8 points) Indicate the appropriate answer for each question in the space provided.

   a) The standard normal distribution has C.
   A) mean = 0, standard deviation = 0 B) mean = 1, standard deviation = 0
   C) mean = 0, standard deviation = 1 D) mean = 1, standard deviation = 1

   b) The location/center of a normal distribution is determined by its B.
   A) standard deviation σ B) mean μ C) interquartile range IQR D) variance σ^2

   c) For a continuous random variable X, the total area under the probability distribution curve of X is always C.
   A) less than 1 B) greater than 1 C) equal to 1 D) π

   d) The A of a discrete random variable gives the possible values of the variable and the associated probabilities.
   A) Probability Distribution B) Mean C) Parameter D) Standard deviation

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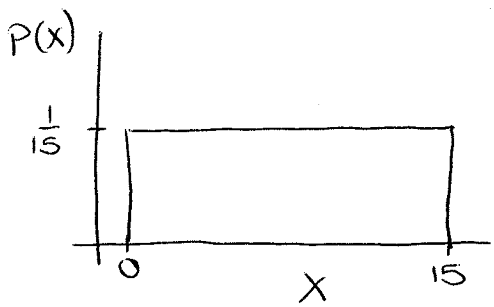
3. (8 points) A life insurance company sells a $150,000 one-year term life insurance policy to a 35-year-old male for $250. If the probability that the male survives the year is 0.998592, compute the expected value of this policy to the insurance company.

		x	$P(x)$
lives		250	.998592
dies		-149750	.001408

$$E(x) = 250(.998592) + (-149750)(0.001408)$$
$$= 249.65 \quad -210.85$$
$$= \$38.80$$

4. (8 points) A random number generator randomly generates a number between 0 and 15. The random variable X, the number generated, follows a uniform probability distribution.
   a) Draw the graph of the uniform density function, and clearly label your axes.



   b) What is the probability of generating a number between 2.5 and 3.7?

$$P(2.5 \leq X \leq 3.7) = (3.7 - 2.5)\tfrac{1}{15} = (1.2)(\tfrac{1}{15})$$
$$= 0.08$$

5. (6 points) For each of the following probability experiments, does the experiment represent a binomial experiment? If yes, give the values for *n* and *p*. If no, state why.
   a) A coin is tossed until 3 heads have appeared. The number to tosses required is recorded.

No - number of trials is not fixed

   b) According to an article in Reader's Digest, 10% of people are left-handed. For 12 people selected at random, the number of people who are left-handed is recorded.

Yes, $n = 12$, $p = 0.1$

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6. The following data represent the number of games played in each World Series from 1923 to 2005.

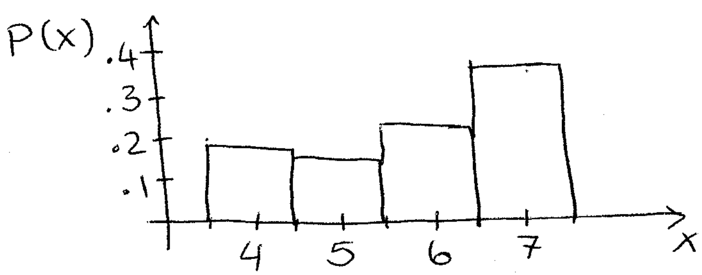
x (games played)	Frequency
4	16
5	15
6	18
7	33

Total = 82

a) (6 points) Construct a discrete probability distribution for the random variable X.

X	P(x)
4	$\frac{16}{82}$ = .195
5	$\frac{15}{82}$ = .183
6	$\frac{18}{82}$ = .220
7	$\frac{33}{82}$ = .402

b) (4 points) Draw a probability histogram for this distribution.



c) (5 points) Compute and interpret the mean of the random variable X.

$\mu_X = 4(.195)+5(.183)+6(.220)+7(.402) = 5.829$

If a Series ~~game~~ is selected at random, it is expected that 6 games will have been played.

d) (5 points) Compute the standard deviation of the random variable X.

X	μ	$X-\mu$	$(x-\mu)^2$	P(x)	$(x-\mu)^2 P(x)$
4	5.829	−1.829	3.345	.195	.652
5	5.829	−.829	.687	.183	.126
6	5.829	.171	.029	.220	.006
7	5.829	1.171	1.371	.402	.551
					1.335

$\sigma_X = \sqrt{1.335} = 1.155$

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7. (15 points) According to the Nielsen Ratings, 30% of households nationwide tuned in to watch the 1981 World Series between the LA Dodgers and the NY Yankees. Suppose a random sample of 15 households is chosen.

a) What is the probability that exactly 5 were tuned into the 1981 World Series?

$n = 15$

$p = 0.3$ $\quad P(5) = .2061$ ✓✓✓

b) What is the probability that 3 or fewer were tuned into the 1981 World Series?

$P(X \leq 3) = .2969$ ✓✓ ✓

c) Find the probability that at least 4 were tuned into the 1981 World Series.

$P(X \geq 4) = 1 - P(X \leq 3) = .7031$ ✓ ✓ ✓

d) What is the mean number of households that were tuned into the 1981 World Series?

$\mu = np = 15(0.3) = 4.5$ ✓ ✓ ✓

e) What is the standard deviation of the number of households that were tuned into the 1981 World Series?

$\sigma = \sqrt{np(1-p)} = \sqrt{15(0.3)(0.7)} = \sqrt{3.15} = 1.77$ ✓ ✓ ✓

8) (15 points) Suppose Z is a standard normal random variable. Find:

a) P(Z < 2.25)

0.9878

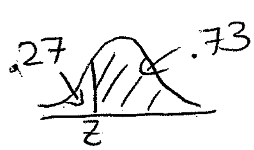
b) P(-1.32 < Z < 1.13)

$.8708 - 0.0934 = .7774$

c) P(Z > -1.99)

$1 - .0233 = .9767$

d) Z such that the area to the right of Z is 0.73.



$z = -0.61$

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9) (15 points) Suppose Wigglesworth Tires manufactures tires having the property that the mileage the tire lasts approximately follows a normal distribution with mean 65,000 miles and standard deviation 3,300 miles.

a) What percent of the tires will last at least 70,000 miles?

$\mu = 65000$

$\sigma = 3300$

$X = 70\,000$

$$Z = \frac{70000 - 65000}{3300} = 1.52$$

$P(X \geq 70000)$

$= P(Z \geq 1.52)$

$= 1 - P(Z < 1.52)$

$= 1 - .9357$

$= 0.0643$

b) What is the probability that a randomly selected tire lasts between 60,000 and 75,000 miles?

$$Z_1 = \frac{60\,000 - 65000}{3300} = -1.52$$

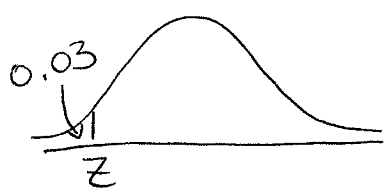
$$Z_2 = \frac{75\,000 - 65000}{3300} = 3.03$$

$P(60\,000 \leq X \leq 75000)$

$= P(-1.52 \leq Z \leq 3.03)$

$= .9988 - 0.0643 = .9345.$

c) Mr. Wigglesworth wants to issue a warranty that will offer to replace any tire lasting less than the guaranteed lifetime. What lifetime should Mr. Wigglesworth advertise on the warranty so that his company will have to replace no more than 3% of its tires?



$Z = -1.88$

$X = Z\sigma + \mu$

$= (-1.88)(3300) + 65000$

$= -6204 + 65000$

$= 58,796.$

THE END

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