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MATH 130 - ELEMENTS OF STATISTICS
EXAM III - VERSION 1

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NAME: Key

SHOW ALL WORK NEATLY and clearly indicate your answers. Show all your working for every problem. A correct answer with no work shown (except on problems which are trivial) will receive no credit. If you are not sure if you have written enough, please ask. There are 7 problems on 5 pages. The time allowed is 75 minutes and the exam is worth 100 points. Write your name on every page in the space provided. You may use a calculator for this test, but not a cell-phone or laptop. You may also use a 3x5 inch formula card, and you may use the tables in the back of your textbook to calculate the probabilities for the normal and Student's t-distributions.

Good luck!!

1. In a 2003 study it was found that medical residents work an average of 81.7 hours per week. Suppose the number of hours worked per week ~~my~~ by medical residents is normally distributed with standard deviation 6.9 hours per week. Suppose a random sample of 8 medical students is chosen.

   (a) (6 points) What is the probability that a randomly selected medical resident works less than ~~72~~ 77 hours per week?

   $$Z = \frac{77 - 81.7}{6.9} = -0.68$$

   $$P(X < 72) = P(Z < -0.68) = .2483$$

   (b) (3 points) What is the mean of the sampling distribution of $\overline{x}$, the mean number of hours worked per week?

   $$\mu_{\bar{x}} = 81.7$$

   (c) (3 points) What is the standard deviation of the sampling distribution of $\overline{x}$?

   $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.9}{\sqrt{8}} = 2.4395$$

   (d) (6 points) What is the probability that the mean number of hours worked per week by the sample of 8 students is less than ~~72~~ 77 hours per week?

   $$Z = \frac{77 - 81.7}{2.4395} = -1.93$$

   $$P(\overline{X} < 77) = P(Z < -1.93) = 0.0268$$

   (e) (3 points) Why do we know that the sampling distribution of $\overline{x}$ follows a normal distribution?

   The population is normally distributed.

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2. Suppose Nokia wishes to estimate the mean talk time on its new CG134 camera phone before the battery must be recharged. In a random sample of 40 phones, the sample mean talk time was 315 minutes.

(a) (3 points) Why can we say that the sampling distribution of $\overline{x}$ is approximately normal?

Sample size > 30

(b) (10 points) Construct a 94% confidence interval for the mean talk time for all CG134 camera phones, assuming that $\sigma = 27$ minutes.

$\alpha = 0.06 \Rightarrow Z_{\alpha/2} = Z_{.03} = 1.88$

Lower bound $= \overline{x} - Z_{.03}\frac{\sigma}{\sqrt{n}} = 315 - 1.88 \cdot \frac{27}{\sqrt{40}} = 315 - 8.03$

$= 306.97$

Upper bound $= 323.03$

CI $= (306.97, 323.03)$

(c) (2 points) Interpret the confidence interval you found in part b).

We are 94% that the mean talk time is between 306.97 & 323.03 minutes.

(d) (2 points) What would happen to the length of the confidence interval that you found in part b) if it had been instead calculated with a sample of 60 phones? (You do not need to find the confidence interval.)

The interval would have decreased in length.

(e) (6 points) How many phones would Nokia need to test to estimate the mean talk time for all CG134 phones within 5 minutes with 98% confidence (assuming that $\sigma = 27$ minutes)?

$n = \left(\frac{Z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{2.33 \cdot 27}{5}\right)^2 = 158.31$

$n = 159$

$\alpha = 0.02$

$Z_{\alpha/2} = Z_{.01} = 2.33$

$E = 5$

$\sigma = 27$

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3. (6 points) For the Student's t-distribution, find the t-value so that

   (a) the area in the right tail is 0.005 with 19 degrees of freedom.

   $t = 2.861$

   (b) the area left of the t-value is 0.025 with 12 degrees of freedom.

   $t = -2.179$

4. A researcher records the number of cars that arrive at a Burger King drive-through between 11:50am and 12:00noon for a random sample of 25 Wednesdays. The researcher finds that the sample mean is 4.08 and the sample standard deviation is 2.12.

   (a) (12 points) Compute *and interpret* a 90% confidence interval for the mean number of cars waiting in line between 11:50am and 12:00noon on Wednesdays.

   $\bar{x} = 4.08$ $s = 2.12$ $n = 25$ $\alpha = .1$

   $t_{\alpha/2} = t_{.05} = 1.711$ (with 24 df).

   $t_{\alpha/2}\frac{s}{\sqrt{n}} = 1.711 \cdot \frac{2.12}{\sqrt{25}} = .7255$

   Lower bound $= \bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} = 4.08 - .7255 = 3.3545$

   Upper bound $= 4.8055$

   We are 90% confident the mean number of cars arriving is between 3.35 & 4.81

   (b) (2 points) What would happen to the length of the above interval if it were calculated with 95% confidence? (You do not need to find the confidence interval.)

   The length of the interval would increase

5. (3 points) Which is larger, the area under the t−distribution with 12 degrees of freedom to the right of $t = 2.5$ or the area under the standard normal distribution to the right of $z = 2.5$?

   t-distribution.

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6. From a random sample of 1201 Americans, it was discovered that 1139 of them lived in neighborhoods with acceptable levels of carbon monoxide.

   (a) (4 points) Obtain a point estimate for the proportion of Americans who live in neighborhoods with acceptable levels of carbon monoxide.

$$\hat{p} = \frac{1139}{1201} = .9484$$

   (b) (8 points) Construct a 99% confidence interval for the proportion of Americans who live in neighborhoods with acceptable levels of carbon monoxide.

$\alpha = .01$ $\quad \frac{\alpha}{2} = .005$ $\quad Z_{.005} = 2.575$ $\quad n = 1201$

$$Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 2.575\sqrt{\frac{.9484(1-.9484)}{1201}} = 0.0164$$

Upper bound = .9484 + .0164 = .9648

Lower bound = .9484 − .0164 = .932

   (c) (10 points) You wish to conduct your own study to determine the proportion of Americans who live in neighborhoods with acceptable levels of carbon monoxide.

      i. What sample size would be needed for the estimate to be within 2 percentage points with 95% confidence if you use as a prior estimate your answer from part ~~b~~ a)

$\alpha = .05$

$Z_{\alpha/2} = Z_{.025} = 1.96$

$E = .02$

$$n = \hat{p}(1-\hat{p})\left(\frac{Z_{\alpha/2}}{E}\right)^2$$
$$= .9484(1-.9484)\left(\frac{1.96}{.02}\right)^2$$
$$= 469.99$$

$n = 470$

      ii. What sample size would be needed for the estimate to be within 2 percentage points with 95% confidence if you do not use a prior estimate?

$$n = 0.25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = 0.25(9604) = 2401$$

$n = 2401$

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7. The manufacturer of a certain type of ATM machine reports that the mean ATM withdrawal is \$60. The manager of a convenience store with an ATM machine claims that the mean withdrawal from his machine is less than this amount.

   (a) (6 points) Determine the null and alternative hypotheses.

   $H_0: \mu = 60$

   $H_1: \mu < 60$

   (b) (3 points) What would it mean to make a Type I error?

   Reject H_0 when H_0 is true

   Evidence leads the manager to believe that the mean withdrawal is less than \$60, when in fact it is not.

   ✓

   (c) (3 points) What would it mean to make a Type II error?

   Fail to reject H_0 when H_1 is true

   The manager does not reject the null hypothesis when, in fact, the mean withdrawal is less than \$60.

   ✓

   (d) (3 points) Suppose sample data indicates that the null hypothesis should be rejected. State the conclusion of the manager.

   There is sufficient evidence to support that the mean withdrawal is less than \$60.

THE END

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