

MATH 130.05: Elements of Statistics I

Quiz 3 (15 points)
November 17, 2009

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Answer all questions. Show all your working neatly and indicate your answers clearly.
(Recall: $\mu_{\bar{x}} = \mu$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, $s_{\bar{x}} = \frac{s}{\sqrt{n}}$. Confidence interval bounds are $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ (σ known)
and $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$ (σ unknown).)

1. (5 points) Suppose a simple random sample of size $n = 35$ is obtained from a normally distributed population with $\mu = 72$ and $\sigma = 13$.

(a) What is the mean of the sampling distribution of \bar{x} ?

$$\mu_{\bar{x}} = \mu = 72$$

(b) What is the standard deviation of the sampling distribution of \bar{x} ?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{13}{\sqrt{35}} = 2.197$$

(c) What is the probability that the random sample has a mean of 70 or less?

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{70 - 72}{2.197} = -.91$$

$$P(\bar{x} \leq 70) = P(z < -.91) = .1814$$

2. (4 points) Based on a random sample of 1300 individuals, the mean amount of time spent eating or drinking each day is 1.21 hours. Assuming the population standard deviation for time spent eating or drinking is 0.65 hours, construct and interpret a 90% confidence interval for the mean amount of time spent eating or drinking each day.

$$\bar{x} = 1.21, \quad n = 1300, \quad \sigma = .65 \quad \alpha = .1 \quad \alpha/2 = .05$$

$$z_{\alpha/2} = z_{.05} = 1.645, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.65}{\sqrt{1300}} = 0.018$$

$$\text{Lower bound} = 1.21 - 1.645(0.018) = 1.21 - .0296 = 1.18$$

$$\text{Upper bound} = 1.21 + .0296 = 1.24$$

We are 90% confident that the mean time spent eating or drinking each day is between 1.18 & 1.24 hours.

THERE ARE MORE QUESTIONS ON THE OTHER SIDE OF THE PAPER!!

3. (2 points) For the Student's t -distribution, find the t -value so that

(a) the area in the right tail is 0.15 with 23 degrees of freedom.

$$1.060$$

(b) the area left of the t -value is 0.005 with 6 degrees of freedom.

$$-3.707$$

4. (4 points) A survey of 39 individuals asked "How many books did you read in the last year?" Results of the survey indicated that $\bar{x} = 18.5$ books and $s = 12$ books. Construct and interpret a 98% confidence interval for the mean number of books read during the last year. (You may assume that the population number of books read is normally distributed.)

$$n=39, \bar{x}=18.5, S_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{12}{\sqrt{39}} = 1.922$$

$$\alpha = .02 \quad t_{\alpha/2} = t_{.01} = 2.429 \quad (\text{with } 38 \text{ df})$$

$$\begin{aligned} \text{Lower bound} &= \bar{x} - t_{\alpha/2} (S_{\bar{x}}) = 18.5 - 2.429(1.922) = 18.5 - 4.67 \\ &= 13.8 \end{aligned}$$

$$\text{Upper bound} = 23.2$$

We are 98% confident that the mean number of books read during the last year is between

13.8 and 23.2.