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MATH 211.01 - CALCULUS II
EXAM III

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December 8, 2009

NAME: Key

SHOW ALL WORK NEATLY and clearly indicate your answers. Show all your working for every problem. A correct answer with no work shown (except on problems which are trivial) will receive no credit. If you aren't sure if you have written enough, please ask. There are 10 problems on 4 pages. The time allowed is 50 minutes and the exam is worth 100 points. *Put your name on each page.* If you need extra space, complete the problem on the back of the page and indicate that you have done so. GOOD LUCK!!

1. (a) (6 points) Set up the integral to calculate the arc length of the curve described by the parametric equations $\begin{cases} x = \sin(9t) \\ y = t^2 - 1 \end{cases}$, $-\pi \le t \le \pi$. *You do not need to evaluate the integral.*

$x' = 9\cos 9t, \quad y' = 2t$

$$\therefore S = \int_{-\pi}^{\pi} \sqrt{(9\cos 9t)^2 + (2t)^2}\, dt$$

(b) (8 points) Set up the integral to calculate the surface area formed when the curve described by the parametric equations $\begin{cases} x = t^3 - 6t \\ y = t^4 - 6t \end{cases}$, $-1 \le t \le 1$, is revolved around the line $y = -4$. *You do not need to evaluate the integral.*

$x' = 3t^2 - 6, \quad y' = 4t^3 - 6$

$$\therefore S = \int_{-1}^{1} 2\pi \left| (t^4 - 6t) - (-4) \right| \sqrt{(3t^2-6)^2 + (4t^3-6)^2}\, dt$$

2. (8 points) Find the slope of the tangent line to the curve $\begin{cases} x = 3t^3 + 1 \\ y = 2t^2 - 4 \end{cases}$ at the point $(4, -2)$.

$3t^3 + 1 = 4$ if $t = 1$

Check: $2(1)^2 - 4 = -2$. So $t = 1$.

$x' = 9t^2 \qquad y' = 4t$

$$\therefore \left.\frac{dy}{dx}\right|_{t=1} = \frac{4(1)}{9(1)^2} = \frac{4}{9}.$$

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3. (a) (4 points) Find rectangular coordinates for the polar point $\left(7, \frac{\pi}{6}\right)$.

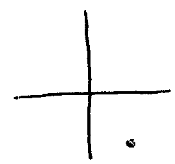
$x = 7\cos\frac{\pi}{6} = 7 \cdot \frac{\sqrt{3}}{2} = \frac{7\sqrt{3}}{2}$

$y = 7\sin\frac{\pi}{6} = 7 \cdot \frac{1}{2} = \frac{7}{2}$

$\therefore (x, y) = \left(\frac{7\sqrt{3}}{2}, \frac{7}{2}\right)$

(b) (6 points) Find all polar coordinate representations of the rectangular point $(3, -8)$.

$r^2 = 3^2 + (-8)^2 = 9 + 64 = 73$ ✓ $\therefore r = \pm\sqrt{73}$



$\theta = \tan^{-1}\left(-\frac{8}{3}\right) = -1.212$ or 1.930

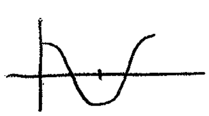
$\therefore (r, \theta) = (\sqrt{73}, -1.212) = (-\sqrt{73}, 1.930)$.

4. (15 points) Find the area of the region inside $r = 1 + \cos\theta$.

Full picture for: $0 \le \theta \le 2\pi$ (or $-\pi \le \theta \le \pi$).

$1 + \cos\theta = 0$

$\cos\theta = -1$



$\theta = \cancel{-\frac{\pi}{2}}, \cancel{\frac{3\pi}{2}} -\pi, \pi$

$\therefore A = \int_0^{2\pi} \frac{1}{2}[1 + \cos\theta]^2 \, d\theta$

$= \int_0^{2\pi} \frac{1}{2}(1 + 2\cos\theta + \cos^2\theta) \, d\theta$

$= \int_0^{2\pi} \frac{1}{2} + \cos\theta + \frac{1}{4}(1 + \cos 2\theta) \, d\theta$

$= \int_0^{2\pi} \frac{3}{4} + \cos\theta + \frac{1}{4}\cos 2\theta \, d\theta$

$= \frac{3}{4}\theta + \sin\theta + \frac{1}{8}\sin 2\theta \Big|_0^{2\pi}$

$= \left(\frac{3}{4}(2\pi) + 0 + 0\right) - \left(\frac{3}{4}(0) + 0 + 0\right)$

$= \frac{3\pi}{2}$.

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5. (8 points) Find parametric equations for the line segment from $(1, -7)$ to $(4, 3)$.

$x(t) = 1 + 3t$

$y(t) = -7 + 10t \quad , \quad 0 \leq t \leq 1$

6. (8 points) Find an $x-y$ equation corresponding to the curve given by the parametric equations $\begin{cases} x &= 3 - t \\ y &= t^2 + 2t \end{cases}$

$t = 3 - x$

$\therefore \; y = (3-x)^2 + 2(3-x)$

7. (12 points) The position of an object is given by the parametric equations $\begin{cases} x &= 5\cos t + \sin 5t \\ y &= 5\sin t + \cos 5t \end{cases}$. Find the object's velocity and speed at time $t = 0$.

$x' = -5\sin t + 5\cos 5t$

$x'(0) = -5(0) + 5(1) = 5$

$y' = 5\cos t - 5\sin 5t$

$y'(0) = 5(1) - 5(0) = 5$

Velocity is 5 upward and 5 to the right.

Speed $= \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

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8. (10 points) Find the slope of the tangent line to the curve $r = \sin(4\theta)$ at $\theta = \frac{\pi}{4}$.

$x = \sin 4\theta \cos\theta$ $\qquad$ $y = \sin 4\theta \sin\theta$

$x' = 4\cos 4\theta \cos\theta - \sin 4\theta \sin\theta$ $\qquad$ $y' = 4\cos 4\theta \sin\theta + \sin 4\theta \cos\theta$

$x'(\frac{\pi}{4}) = 4(-1)(\frac{1}{\sqrt{2}}) - 0 = -2\sqrt{2}$ $\qquad$ $y'(\frac{\pi}{4}) = 4(-1)\frac{1}{\sqrt{2}} + 0 = -2\sqrt{2}$

$\therefore$ Slope of tangent line $= \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{y'(\frac{\pi}{4})}{x'(\frac{\pi}{4})} = \frac{-2\sqrt{2}}{-2\sqrt{2}} = 1.$

9. (6 points) Find a polar equation corresponding to the rectangular equation $y^2 - x^2 = 25$.

$(r\sin\theta)^2 - (r\cos\theta)^2 = 25$

$r^2(\sin^2\theta - \cos^2\theta) = 25$

$r^2 = \frac{25}{\sin^2\theta - \cos^2\theta}$

$r = \pm\frac{5}{\sqrt{\sin^2\theta - \cos^2\theta}}$

10. (10 points) Sketch the graph of $r = 3 - 6\cos\theta$ and identify all values of θ where $r = 0$ and a range of values of θ that produces one copy of the graph. Show all working as a graph with no working shown will receive no credit.

$r = 3 - 6\cos\theta$

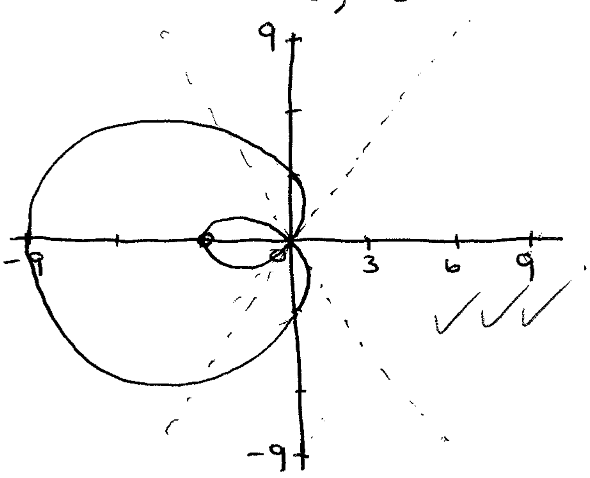
$0 = 3 - 6\cos\theta$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Complete graph for $0 \le \theta \le 2\pi$

Interval	$\cos\theta$	$-6\cos\theta$	$3-6\cos\theta$
$(0, \frac{\pi}{3})$	$1 \searrow \frac{1}{2}$	$-6 \nearrow -3$	$-3 \nearrow 0$
$(\frac{\pi}{3}, \frac{\pi}{2})$	$\frac{1}{2} \searrow 0$	$-3 \nearrow 0$	$0 \nearrow 3$
$(\frac{\pi}{2}, \pi)$	$0 \searrow -1$	$0 \nearrow 6$	$3 \nearrow 9$
$(\pi, \frac{3\pi}{2})$	$-1 \nearrow 0$	$6 \searrow 0$	$9 \searrow 3$
$(\frac{3\pi}{2}, \frac{5\pi}{3})$	$0 \nearrow \frac{1}{2}$	$0 \searrow -3$	$3 \searrow 0$
$(\frac{5\pi}{3}, 2\pi)$	$\frac{1}{2} \nearrow 1$	$-3 \searrow -6$	$0 \searrow -3$



THE END

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