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MATH 310.01 - METHODS OF PROOF
EXAM I

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September 25, 2009

NAME: Key

SHOW ALL WORK NEATLY and clearly indicate how you arrived at your answers. If you aren't sure if you have written enough, please ask. There are 10 problems on 5 pages. The time allowed is 50 minutes and the exam is worth 100 points. *Put your name on each page.* If you need extra space, complete the problem on the back of the page and indicate that you have done so. GOOD LUCK!

1. (10 points) Construct a truth table for $(Q \wedge \sim P) \vee (P \Rightarrow Q)$. Show your work, including the truth values for all the intermediate expressions.

P	Q	$\sim P$	$Q \wedge \sim P$	$P \Rightarrow Q$	$(Q \wedge \sim P) \vee (P \Rightarrow Q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	F	T	T

2. (6 points) Suppose the statement "If Izzy likes catnip or she does not like fish then she is not a cat" is false, and the statement "Izzy likes catnip" is false. What is the truth value of the statement "Izzy likes fish"?

P: Izzy likes catnip
Q: She does not like fish
R: She is not a cat

$(P \vee Q) \Rightarrow R$ is false. ~~and P is false~~

So $P \vee Q$ is true, but P is false,

so Q is true

Izzy likes fish (ie: $\sim Q$) is false.

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3. (20 points) Let $\mathcal{U} = \{n \in \mathbb{N} | 1 \leq n \leq 10\}$, and let $A = \{1, 3, 5\}$, $B = \{2, 3, 4, 5, 8, 10\}$, and $C = \{3, 6, 9\}$. Use the roster method to list the elements of each of the following sets. Write your answer in the space provided.

(a) $A \cup C =$ $\{1, 3, 5, 6, 9\}$

(b) $A \cap B =$ $\{3, 5\}$

(c) $\overline{B} =$ $\{1, 6, 7, 9\}$

(d) $C - B =$ $\{6, 9\}$

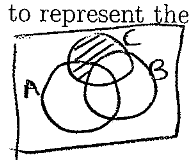
(e) $\mathcal{P}(C) =$ $\{\phi, \{3\}, \{6\}, \{9\}, \{3,6\}, \{3,9\}, \{6,9\}, \{3,6,9\}\}$

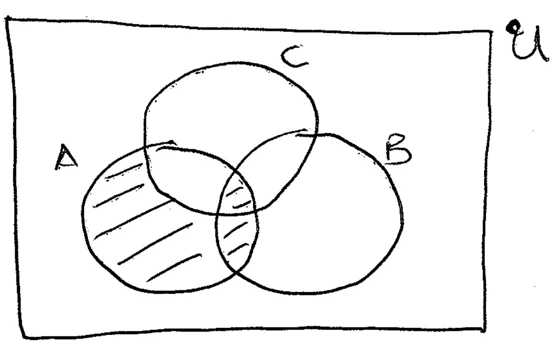
(f) $|C| =$ 3

(g) $A \times C =$ $\{(1,3), (3,3), (5,3), (3,6), (1,6), (1,9), (3,9), (5,6), (5,9)\}$

4. (8 points) Let A, B and C be subsets of some universal set $\mathcal{U}$. Draw a Venn diagram to represent the region $A - (\overline{B} \cap C)$.

$\overline{B} \cap C$:





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5. (15 points) Consider the following statements:
   P:Cassie is black,
   Q:Cassie is a cat,
   R:Cassie is friendly.

   (a) Express the statement $(\sim P) \Rightarrow (Q \wedge R)$ as an English sentence.

   If Cassie is not black then she is a cat and she is friendly.

   (b) Write the following sentence using logical connectives and the component statements as named above: "Cassie is a cat only if she is black and friendly."

   $Q \Rightarrow (P \wedge R)$

   (c) Negate the statement "If Cassie is not friendly then she is not black." Write your answer as an English sentence.

   $$\sim(\sim R \Rightarrow \sim P) \equiv \sim(\sim(\sim R) \vee (\sim P))$$
   $$\equiv \sim R \wedge P$$

   Cassie is not friendly and she is black

6. (6 points) Use set-builder notation to describe the set of all natural numbers that are strictly between 7 and 42.

   $\{n \in \mathbb{N} : 7 < n < 42\}$

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7. (12 points) Consider the open sentence $P(x, y) : xy = 36$ over the set $S = \{1, 2, 3, 4, 6, 9\}$.

(a) What is the truth value of $\forall x, y \in S, P(x, y)$? False

(b) What is the truth value of $\exists x \in S, \forall y \in S, P(x, y)$? False

(c) What is the truth value of $\forall x \in S, \exists y \in S, P(x, y)$? ~~True~~ False

(d) Give the truth set of $\exists x \in S, P(x, y)$.

~~$\{1, 2, 3, 4, 6, 9\}$~~

$\{4, 6, 9\}$

8. (10 points) Use established logical equivalences to prove that $P \Rightarrow (Q \vee R)$ is logically equivalent to $(P \wedge \sim R) \Rightarrow Q$.

$$
\begin{aligned}
(P \wedge \sim R) \Rightarrow Q &\equiv \sim(P \wedge \sim R) \vee Q \\
&\equiv (\sim P \vee \sim(\sim R)) \vee Q \\
&\equiv (\sim P \vee R) \vee Q \\
&\equiv \sim P \vee (R \vee Q) \\
&\equiv \sim P \vee (Q \vee R) \\
&\equiv P \Rightarrow (Q \vee R)
\end{aligned}
$$

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9. (6 points) Let $A_i = \{i, 2i, 3i\}$ for $i = 1, 2, 3, 4$. Determine each of the following:

   (a) $\cup_{i=1}^{4} A_i$

   $A_1 = \{1, 2, 3\}$

   $A_2 = \{2, 4, 6\}$

   $A_3 = \{3, 6, 9\}$

   $A_4 = \{4, 8, 12\}$

   $\bigcup_{i=1}^{4} A_i = \{1, 2, 3, 4, 6, 8, 9, \cancel{6}, 12\}$

   (b) $A_1 \cap A_3$

   $\{3\}$

10. (10 points) Prove that if x is an even integer and y is an odd integer then $2x + 5y$ is an odd integer.

Let x, y be integers such that x is even & y is odd.

Then there exist integers k, j such that

$x = 2k$ and $y = 2j + 1$.

So
$$2x + 5y = 2(2k) + 5(2j+1)$$
$$= 4k + 10j + 5$$
$$= 2(2k + 5j + 2) + 1$$

As $2k + 5j + 2$ is an integer,

$2x + 5y$ is odd.

THE END

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