MATH 310.01 - METHODS OF PROOF
HOMEWORK 9 KEY

Chapter 8:13, 18, 25
Due: November 16, 2009

Question 8.13

A relation R is defined on $\mathbf{Z}$ by xRy if $xy \geq 0$. Prove or disprove:

1. **R is reflexive:**
   This statement is true. Let $a \in \mathbf{Z}$. Then $a \cdot a = a^2 \geq 0$, so aRa and we have that R is reflexive.

2. **R is symmetric:**
   This statement is true. Let $a, b \in \mathbf{Z}$ and suppose that aRb. Then $ab \geq 0$. But $ba = ab$, so $ba \geq 0$. Thus bRa, and we have that R is symmetric.

3. **R is transitive:**
   This statement is false. Consider $a = -1$, $b = 0$ and $c = 1$. Then $ab = (-1)(0) = 0 \geq 0$ so $(-1)R0$, and $bc = (0)(1) = 0 \geq 0$ so $0R1$, but $ac = (-1)(1) = -1 \not\geq 0$, so $-1 \not R 1$. Thus R is not transitive.

Question 8.18

Let $A = \{1, 2, 3, 4, 5, 6\}$. The distinct equivalence classes resulting from an equivalence relation R on A are $\{1, 4, 5\}$, $\{2, 6\}$ and $\{3\}$. What is R?

$R = \{(1, 1), (1, 4), (1, 5), (4, 1), (4, 4), (4, 5), (5, 1), (5, 4), (5, 5), (2, 2), (2, 6), (6, 2), (6, 6), (3, 3)\}$

Question 8.25

A relation R is defined on $\mathbf{Z}$ by xRy if $3x - 7y$ is even. Prove that R is an equivalence relation and determine the distinct equivalence classes.

- R is reflexive: Let $x \in \mathbf{Z}$. Then $3x - 7x = -4x = 2(-2x)$. As $-2x \in \mathbf{Z}$, we have that $3x - 7x$ is even, and so xRx. Thus R is reflexive.

- R is symmetric: Let $x, y \in \mathbf{Z}$ such that xRy. Then $3x - 7y$ is even. Thus there exists and integer k such that $3x - 7y = 2k$. Now $3y - 7x = (-4x - 4y) - (3x - 7y) = 2(-2x - 2y - k)$. As $-2x - 2y - k \in \mathbf{Z}$, $3y - 7x$ is even, and so yRx. Thus R is symmetric.

- R is transitive: Let $x, y, z \in \mathbf{Z}$ such that xRy and yRz. So $3x - 7y$ is even and $3y - 7z$ is even. Then there exist integers k, j such that $3x - 7y = 2k$ and $3y - 7z = 2j$. Now $3x - 7z = (3x - 7y) + (7y - 3y) + (3y - 7z) = 2k + 4y + 2j = 2(k + 2y + j)$. As $k + 2y + j$ is an integer, we have that $3x - 7z$ is even, and xRz. Thus R is transitive.

- As R is reflexive, symmetric and transitive, we have that R is an equivalence relation.

- Equivalence classes: The distinct equivalence classes are the set of even integers and the set of odd integers.