

MATH 464.01 – REAL ANALYSIS I SPRING 2010

- Instructor:** Dr. Antonia Cardwell
- Office:** 206B Wickersham
- Phone:** x3479 (You can leave a message on my voice mail if I'm not in, but e-mail is better.)
- E-mail:** Antonia.Cardwell@millersville.edu
- Website:** <http://www.millersville.edu/~cardwell/spring10/464-01.html>
- Lecture Hours:** 12:00 - 12:50 MWF, 218 Wickersham Hall
- Office Hours:** 10:00-10:50 TR; 11:00-11:50 MF; 1:00-1:50 W
If the above times are inconvenient, see me after class and we'll make an appointment. I am also around at other times, so feel free to drop in.
- Text:** *"Elements of Real Analysis"*, by Charles G. Denlinger (Pre-publication manuscript, January 2010, Jones and Bartlett Publishers) - Available at MU Copy Shop
- Description:** In this course we attempt a rigorous development of the theory of calculus from first principles. We give careful definitions of terms, and correct proofs of theorems. For more detail see the "Course Outline" and "Course Objectives" later in this syllabus. Prerequisites for this course are MATH 310 (220), MATH 322 (242), and MATH 311 (261).
- Grading:** Your grade will be calculated based on the following work:
- | | |
|-------------------------------------|------------|
| 2 in-class exams (100 points each): | 200 |
| Final exam: | 150 |
| Homework: | 100 |
| Total points: | <u>450</u> |
- Homework:** Homework will be assigned daily and is an essential part of the course. I will specify which of the problems I would like you to hand in to be graded, and the graded problems will be collected weekly (typically on Friday.) For homework that is handed in after the due date, one point will be deducted for each class day that the homework is late. The problems to be collected in will be announced in class and posted on the course website (see above.)
- Exams:** There will be two 100-point in-class exams and one 150-point final exam. The final exam for this section will be held on Wednesday, May 5th, 2010, from 12:30pm - 2:30pm. You will need a *documented* excuse to take a test at any time other than the scheduled time. If possible, contact me ahead of time so that alternative arrangements can be made. Tests will not be rescheduled to accommodate vacation travel - be sure to make your travel arrangements so that they do not conflict with exams, class times or deadlines.

Course Outline:

- < The Real Number System
 - o Fields
 - o Ordered Fields
 - o Natural numbers, integers, and rational numbers in ordered fields
 - o Archimedean ordered fields; density of the rationals and irrationals
 - o The completeness property
 - o Uniqueness of the complete ordered field (the real number system)
- < Sequences
 - o Basic concepts; definition of convergence and limit
 - o Algebra of limits of sequences
 - o Inequalities and limits of sequences

- Divergence to infinity
- Monotone sequences; the monotone convergence theorem and applications
- Cantor's nested intervals theorem
- Subsequences and cluster points; Bolzano-Weierstrass theorem
- Cauchy sequences
- < Topology of the Real Number System
 - Neighborhoods and open sets
 - Interior, exterior and boundary of sets
 - Closed sets and cluster points; Bolzano-Weierstrass theorem for sets
 - Closure of a set
- < Limits of Functions
 - Definitions and examples
 - Algebra of limits of functions; inequalities and limits of functions
 - One-sided limits
 - Infinity in limits
- < Continuous Functions
 - Continuity of a function at a points; definitions and examples
 - Discontinuities; one-sided continuity; monotone functions
 - Continuity on compact sets; the max/min theorem
 - Continuity on intervals; the intermediate value theorem and applications
 - Uniform continuity
- < Differentiable Functions
 - Definitions and examples; differentiability implies continuity
 - Rules for differentiation
 - Relative extrema and monotone functions
 - Intermediate value property of derivatives
 - Rolle's theorem, the mean value theorem, and applications
 - Taylor's theorem (optional)
- < The Riemann Integral
 - Definition of the Riemann integral, using upper and lower Darboux sums
 - Riemann's criterion and Cauchy's criterion for integrability
 - The integral as a limit of Riemann sums
 - Methods of calculating a Riemann integral from its definition
 - Basic existence and additivity theorems (very briefly)
 - Algebraic properties of the integral (very briefly)
 - The fundamental theorem of calculus, first form and second form

Course Objectives:

Upon successful completion of this course, a student will have:

1. Developed an ability to read and comprehend expository mathematical writing in real analysis at the upper undergraduate level.
2. Acquired a comprehensive understanding of the basic concepts of real analysis, including
 - a. Its axiomatic/deductive organization, and flow of ideas.
 - b. Its foundations in the real number system (the complete ordered field).
 - c. The fundamental notions of general topology and their uses in real analysis.
 - d. The theory of continuous real-valued functions of a real variable.
 - e. The theory of differentiable real-valued functions of a real variable.
 - f. The theory of the Riemann integral of real-valued functions of a real variable.
 - g. The fundamental theorem of calculus.
3. Sharpened his/her ability to critique mathematical ideas with constructive skepticism. In particular, will have developed an intellectual storehouse of instances in which naïve intuition would lead him/her to mistakenly believe untrue statements in analysis or disbelieve true statements.
4. Learned that mathematical proof is the one indispensable tool for separating truth from falsehood in analysis.

5. Learned the essential role played by definitions in the formulation of methodology and proofs in mathematics.
6. Developed the specialized techniques of real analysis that allow the above objectives to be realized.
7. Learned significant applications of the concepts and techniques of real analysis.
8. Learned the fundamental role played by real analysis in providing rigorous justification of the methods taught in elementary calculus courses.

University Policies:

Attendance - University Class Attendance Policy (revised SP03):

1. Students are expected to attend all classes. It is the student's responsibility to complete all course requirements even if a class is missed. If a student misses class for an officially excused reason, then he/she is entitled to make up the missed work but only at the convenience of the faculty member. Responsibility for materials presented in, assignments made for, and tests/quizzes given in regularly scheduled classes lies solely with the student.
2. The University policy is that faculty will excuse absences for the following reasons: personal illness; death or critical illness in the family; participation in a university-sponsored activity; jury duty; military duties; religious holidays.
3. Faculty judge the validity of student absences from class within the University's approved guidelines and may require documentation for excused absences. Faculty will evaluate any reason, other than those listed above, for a student missing class and determine whether the absence is justified. In these circumstances, a student may make up missed work at the discretion of the instructor.
4. In the case of foreseeable absences, students are encouraged to notify the faculty member in advance. A student who will miss class due to participation in an official University activity must notify the instructor well in advance of the activity to assure that the absence is excused.

Appeals: As with any academic issue, students may exercise their right to appeal adverse attendance decisions. Please refer to the current undergraduate catalog for the complete Academic Appeal procedure.

Academic Honesty/Integrity: The University's policies regarding academic honesty can be found at <http://muweb.millersville.edu/~govern/sect3/acaddis.html>. Students should observe these policies as I will enforce them.

Special Accommodations: It is University policy to provide reasonable accommodations to students with disabilities. Please contact the Learning Services Office, 348 Lyle Hall, 717-872-3178, to discuss accommodation needs.