

Double Bubble Experiments in the Three-Torus

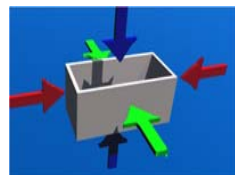
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Sponsored by Dr. Ron Umble, Millersville University and
Dr. Frank Morgan, Williams College

The Problem

Everyone is familiar with the spherical shape of floating soap bubbles. This shape is a consequence of the fact that of all the ways of enclosing a given volume, a sphere has the least surface area (or equivalently, the lowest energy).

In 2000, mathematicians Michael Hutchings, Frank Morgan, Manuel Ritoro, and Antonio Ros announced a proof that the surface area minimizing configuration enclosing two volumes consists of two spheres joined together to form a "standard double bubble".

Imagine yourself in a magical room whose ceiling and floor are mysteriously connected, as are the opposite walls. If you tossed a ball against the right wall, it would disappear and emerge from the left wall. If you looked at the ceiling, you'd see your feet. This room is a model of a four dimensional mathematical object called a three-torus.



Consider the problem of enclosing two volumes with soap bubbles inside such a space. If the bubbles are small in comparison to the room, it's reasonable to suppose that the standard double bubble configuration would be optimal. But if you created bubbles large enough to touch the magical walls, interesting things might happen.

In 2001, Miquel Carrion Alvarez, Joseph Corneli, Genevieve Walsh, and Shabnam Beheshli used calculations and computational simulation to conjecture that every surface area minimizing double bubble configuration in the three-torus is one of the ten types shown in figure 1. This conjecture has not been proven however.

We set out to construct actual examples of as many of the ten configurations as possible in a physical model of the three-torus consisting of a Plexiglas box. Because we don't actually live in four spatial dimensions, our box is only an approximation. A number of the full configurations are likely to be unstable in three dimensions, and in those cases, we inferred the existence of the configuration by the application of reflections.

Results

Considerable experimentation (and some research) was needed to arrive at a satisfactory recipe for the bubble blowing solution and the techniques for creating the bubbles.

We found that a mixture of one part Joy brand dishwashing soap, two parts glycerin, and three parts water produced bubbles that lasted long enough to be photographed.



The Standard Double Bubble

The "standard" double bubble was most easily produced by inverting the box and blowing two bubbles in close proximity. This is an example of the necessity of employing reflection to obtain the complete configuration—the spherical shape results from reflecting the two half-spheres through the base of the box.



The Delaunay Chain

The Delaunay Chain was constructed by blowing a small half bubble on the floor of the box in the middle. Blowing a bubble on either side large enough to contact the walls produced the configuration above. Note that because the left and right walls are identified, the bubbles in contact with those walls are topologically a single bubble.

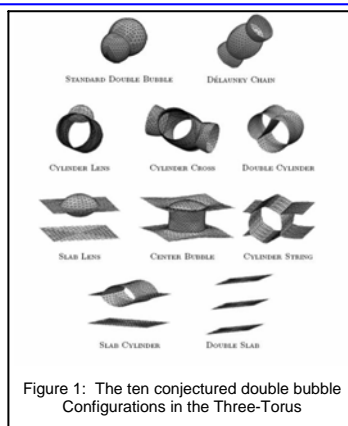


Figure 1: The ten conjectured double bubble Configurations in the Three-Torus



The Cylinder Lens

The Cylinder Lens was obtained by pulling the straw used to blow the bubbles across the width of the box, thus stretching a semi-cylindrical bubble from wall to wall. Reflection in the floor of the box produces a cylinder. Because a small bubble placed on top of the cylinder is unstable without additional support, the "lens" part of this configuration was built against the identified walls in two pieces.

The Double Cylinder was produced in a similar fashion by stretching two bubbles from wall to wall in close proximity to one another.



The Double Cylinder



The Cylinder Cross

Constructing the Cylinder Cross involved stretching quarter cylinders along two opposing walls of the box. Carefully blowing a small bubble between them on the floor (and making use of reflection) completed the configuration.



The Slab Lens

The flat slab of the Slab Lens was produced by blowing a bubble inside the box large enough to reach all four walls. As with the Cylinder Lens, a single bubble atop the slab was found to be unstable, so two half-lenses were placed in contact with identified walls.



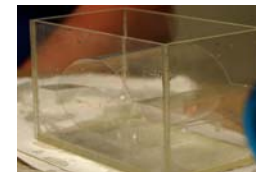
The Center Bubble

The Center Bubble was obtained by blowing a slab as before and then placing the straw on the slab near the middle of a long wall and blowing a bubble that displaces part of the slab. Reflection in the back wall of the box yields the configuration.



The Cylinder String

The Cylinder String was constructed by blowing a semi-cylindrical bubble between the two long walls in the middle of the box. Two parallel quarter cylinders were then placed on either side in contact with the short walls. Reflection in the base of the box completed the configuration.



The Slab Cylinder

One of the more striking configurations is the Slab Cylinder. This was produced by blowing a slab as previously described and then blowing a bubble on top of it in contact with one wall. This bubble was then dragged across to the opposite wall, producing the result shown above.



The Double Slab

A surprisingly challenging configuration (in a box of the size we used—it is much easier to produce in a smaller container) was the Double Slab. First, a slab was blown as previously described. The straw was then pushed through the slab and two bubbles were blown that merged and contacted all four walls. The unwanted membrane between these bubbles was removed by pushing the straw into one of the bubbles and blowing until the membrane ruptured.

Acknowledgments

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