

Research proposal.

Take a polygon P and reflect the plane in (the lines containing) each of its edges. If P has the right properties, each reflection creates new non-overlapping copies of P . Take each polygon in the image and reflect the plane in each of its edges. If P has the right properties, the images of P eventually tessellate the plane. This happens when you start with an equilateral triangle, for example. Try it using Geometer's Sketchpad:

1. Plot two distinct point.
2. Construct the line segment connecting them.
3. Select an endpoint and click [Transform] [Mark Center]
4. Select the segment and both endpoints then click [Transform] [Rotate] [60] degrees [Rotate]
5. Select the rotated endpoint and click [Transform] [Mark Center]
6. Select the rotated segment and both endpoints then click [Transform] [Rotate] [60] degrees [Rotate]

Now you've constructed a perfect equilateral triangle.

1. Select a side and click [Transform] [Mark Mirror].
2. Click [Edit] [Select All].
3. Click [Transform] [Reflect].
4. Repeat steps 1 – 3 until you've tessellated the screen.

Problem: Find all polygons P whose reflections tessellate the plane. Then find, classify and count (equivalence classes) of periodic orbits of a billiard ball in motion on P .

Note: When P is an equilateral triangle, this problem was solved by my former student Andrew Baxter in his senior honors thesis and subsequently appeared in the American Mathematical Monthly, June-July, 2008.

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