Course Syllabus
MATH 205: Geometry for the Middle Level Teacher

Catalog Description with Prerequisites

This course is designed to equip middle level (4-8) mathematics specialists with sufficient knowledge and mathematical experiences for teaching geometry and measurement effectively. The course includes the study of two-dimensional and three-dimensional figures, geometric constructions, congruence, similarity, angle measure, distance, area and volume. Connections between geometry and other mathematics topics, nature and art are addressed.
Prerequisite: passing score on BST, and C or better in MATH 104 or department permission

The need for a geometry course for middle level teachers is justified by the major curricular developments in middle school mathematics. Current recommendations for the middle school mathematics curriculum stress the critical role that middle school plays as a transition point in developing the ideas of geometry and measurement from elementary to secondary grades. Topics to be taught in geometry and measurement in middle school include:

- Develop properties of two- and three-dimensional geometric shapes,
- Promote mathematical arguments proving or disproving geometric relationships among geometric figures,
- Apply transformations and symmetry to analyze mathematical situations,
- Develop spatial reasoning skills and geometric modeling to solve problems,
- Identify measurable quantities, appropriate units and processes of measurement,
- Apply appropriate tools, techniques, and formulas to determine measurements (NCTM, 2000).

Subsequently, the mathematical content preparation for middle level teachers must be of a qualitatively different nature than for elementary and secondary teachers. Currently, the Mathematics Department offers the course "Fundamentals of Mathematics II" [MATH105] which includes units on geometry and measurement. MATH 105 will be retained in its current form, with minor revisions, so that it will better serve the mathematical content needs for Early Childhood [PK – 4] prospective teachers. As a result, while there will be some overlap of topics with MATH 105, there is no comparable course offered from other departments at Millersville University, due to the rigor of the development of the content, and the instructional methods used to teach the course.

The primary focus of the course is to provide students with the content knowledge and learning experiences necessary to teach middle level geometry and measurement as recommended by the Pennsylvania Department of Education *Academic Standards for Mathematics,* the Pennsylvania *Mathematics Assessment Anchors,* and the NCTM *Principles and Standards for School Mathematics*. Students will study the topics of two- and three-dimensional shapes, congruence, and similarity from the multiple perspectives of synthetic, analytic and transformational geometry. Students will study the meaning of fundamental units of measurement and their formulas, determine appropriate units for each attribute, sharpen their skill at using relevant tools and measurement processes, and apply their measurement skills to solve problems. Students will be engaged in learning through completing lab activities using both physical manipulatives [e.g. paper-folding, MIRA] and current dynamic geometry software [e.g. *Geometer's Sketchpad, Geogebra*]. Students will explore the connections between geometry and topics the nature [e.g. symmetry], art and design [e.g. tessellations, wallpaper patterns].

Primary Course Objectives

Upon completion of the course, students will be able to:

Objective	**Forms of Assessment**
1. Apply fundamental rules of logic to form coherent written and oral mathematical arguments	Exams, homework sets and lab exercises
2. Perform standard geometric constructions using paper folding, compass & straightedge, MIRA, and dynamic geometry software	Exams and lab exercises
3. Identify, define, and apply properties of two-and three-dimensional figures	Exams and homework sets
4. Apply rigid motions to the study of congruent figures	Exams, homework sets and lab exercises
5. Apply dilations and other size transformations to the study of similar figures	Exams, homework sets and lab exercises
6. Apply measurement techniques and formulas to geometric figures	Exams, homework sets and lab exercises
7. Demonstrate spatial reasoning skills in two- and three dimensional settings	Lab exercises
8. Demonstrate/apply connections of geometric topics to other disciplines	Lab exercises

In addition to the above course objectives, MATH 205 will help teacher candidates at the Middle Level to fulfill university, state, and national standards as indicated below.

I. In alignment with **Millersville University's Professional Education Unit's Conceptual Framework**, this course will serve to satisfy key indicators for Proficiency Area I: Content Knowledge. Candidates will display knowledge of the mathematical content and apply the important principles and concepts delineated in professional, state, and institutional standards.

1Specifically candidates will:

- Demonstrate competency in their chosen content area.
- Engage in inquiry in their content area that develops their ability to extend student understanding beyond surface information.
- Understand, explain, and apply knowledge of the contextual issues (e.g., political, social, cultural, ethnicity, disability, and gender) that influence their content area.
- Recognize various theories and points of view within their field.
- Develop curricula in a variety of instructional formats reflective of state, national, and local standards.

II. **PDE Standards** that teacher candidates will be better prepared to help their own students to achieve after completing this course include the following:

- PDE 2.3.8 A: Develop formulas and procedures for determining measurements (e.g., area, volume, distance).
- PDE 2.3.8 B: Solve rate problems (e.g., rate × time = distance, principal × interest rate = interest).
- PDE 2.3.8 C: Measure angles in degrees and determine relations of angles.
- PDE 2.3.8 D: Estimate, use and describe measures of distance, rate, perimeter, area, volume, weight, mass and angles.
- PDE 2.3.8 E: Describe how a change in linear dimension of an object affects its perimeter, area and volume.
- PDE 2.3.8 F: Use scale measurements to interpret maps or drawings.
- PDE 2.5.8 A: Justify strategies and defend approaches used and conclusions reached.
- PDE 2.5.8 B: Determine pertinent information in problem situations and whether any further

information is needed for solution.

- PDE 2.9.8 A: Construct figures incorporating perpendicular and parallel lines, the perpendicular bisector of a line segment and an angle bisector using computer software.
- PDE 2.9.8 B: Draw, label, measure and list the properties of complementary, supplementary and vertical angles.
- PDE 2.9.8 C: Classify familiar polygons as regular or irregular up to a decagon.
- PDE 2.9.8 D: Identify, name, draw and list all properties of squares, cubes, pyramids, parallelograms, quadrilaterals, trapezoids, polygons, rectangles, rhombi, circles, spheres, triangles, prisms and cylinders.
- PDE 2.9.8 E: Construct parallel lines, draw a transversal and measure and compare angles formed (e.g., alternate interior and exterior angles).
- PDE 2.9.8 F: Distinguish between similar and congruent polygons.
- PDE 2.9.8 G: Approximate the value of π (pi) through experimentation.
- PDE 2.9.8 H: Use simple geometric figures (e.g., triangles, squares) to create, through rotation, transformational figures in three dimensions.
- PDE 2.9.8 I: Generate transformations using computer software.
- PDE 2.9.8 J: Analyze geometric patterns (e.g., tessellations, sequences of shapes) and develop descriptions of the patterns.
- PDE 2.9.8 K: Analyze objects to determine whether they illustrate tessellations, symmetry, congruence, similarity and scale.

III. Content Knowledge Standard for mathematics teachers from National Council of Teachers of Mathematics for the National Council for the Accreditation of Teacher Educators (2003).

Standard 11: Knowledge of Geometries: Candidates use spatial visualization and geometric modeling to explore and analyze geometric shapes, structures, and their properties.

Indicators

- 11.1 Demonstrate knowledge of core concepts and principles of Euclidean and non-Euclidean geometries in two and three dimensions from both formal and informal perspectives.
- 11.2 Exhibit knowledge of the role of axiomatic systems and proofs in geometry.
- 11.3 Analyze characteristics and relationships of geometric shapes and structures.
- 11.4 Build and manipulate representations of two- and three- dimensional objects and visualize objects from different perspectives.
- 11.5 Specify locations and describe spatial relationships using coordinate geometry, vectors, and other representational systems.
- 11.6 Apply transformations and use symmetry, similarity, and congruence to analyze mathematical situations.
- 11.7 Use concrete models, drawings, and dynamic geometric software to explore geometric ideas and their applications in real-world contexts.
- 11.8 Demonstrate knowledge of the historical development of Euclidean and non-Euclidean geometries including contributions from diverse cultures.

Standard 15: Knowledge of Measurement: Candidates apply and use measurement concepts and tools.

Indicators

- 15.1 Recognize the common representations and uses of measurement and choose tools and units for measuring.
- 15.2 Apply appropriate techniques, tools, and formulas to determine measurements and their application in a variety of contexts.
- 15.3 Completes error analysis through determining the reliability of the numbers obtained from measures.
- 15.4 Demonstrate knowledge of the historical development of measurement and measurement systems including contributions from diverse cultures.

Comprehensive Outline of Course Content

- Logic and deductive reasoning *(2 weeks)*
  a. Mathematical definitions
  b. Types of statements: axiom, theorem, proposition, lemma, corollary, etc.
  c. "If … then" statements; hypothesis, conclusion
  d. Converse, contrapositive, counterexample
  e. Applying the rules of logic
  f. Writing mathematical proofs
- 2-D figures *(4 weeks)*
  a. Definition, properties, and classification of angles, triangles, quadrilaterals, other polygons, and circles
  b. Using coordinate geometry to describe figures
  c. Congruent triangles
  d. Length, angle measure, area: concept and applications
  e. Relevant geometric constructions
- 3-D figures *(3 weeks)*
  a. Definition and properties of polyhedrons, prisms, pyramids, spheres, etc.
  b. Surface area and volume: concept and applications
  c. Relevant geometric constructions
  d. Spatial reasoning
- Rigid motions *(3 weeks)*
  a. Translation, reflection, rotation, and glide reflection
  b. Congruence
  c. Symmetry
  d. Tessellations
  e. Rigid motions in space
  f. Relevant geometric constructions
  g. Symmetry and tessellations in art and design
- Similarity *(3 weeks)*
  a. Dilations and other types of similarities
  b. Similar triangles, polygons, and other figures
  c. Pythagorean theorem
  d. Ratio and proportion
  e. Relevant geometric constructions
- Vectors and Matrices (optional)
  a. Coordinate representation of vectors
  b. Transformations using matrices
- Non-Euclidean geometries (optional)
  a. Taxicab geometry
  b. Spherical geometry
  c. Hyperbolic geometry

Criteria for Evaluating Student Performance

Evaluation of student learning will be accomplished by the following:

- Two or three in-class exams
- A comprehensive final exam
- Lab exercises in and outside of class involving:
  - Compass and straightedge

    - Paper-folding
    - MIRA or other reflective tool
    - Computer software such as *Geometer's Sketchpad* or *Geogebra*
- Homework sets completed outside of class

Textbooks:

- Beem, John K. (2006). *Geometry Connections: Mathematics for Middle School Teachers.* Upper Saddle River, NJ: Pearson Prentice Hall.
- Aichele, Douglas B. and Wolfe, John (2008). *Geometric Structures: An Inquiry-Based Approach for Prospective Elementary and Middle School Teachers.* Upper Saddle River, NJ: Pearson Prentice Hall.

General Education Credit

This course will satisfy G2 credit for MATH.

Resources

- Connected Mathematics Project (CMP). Pearson Education, Inc/Michigan State University. http://connectedmath.msu.edu/.
- Geometric Structures for Elementary Teachers (GeoSET). Oklahoma State University. http://www.math.okstate.edu/geoset/.
- Mathematics in Context (MiC). Encyclopedia Brittanica/University of Wisconsion. http://showmecenter.missouri.edu/showme/mic.shtml.
- MathScape. Glencoe/McGraw-Hill/Educational Development Center. http://www2.edc.org/mathscape/.
- MathThematics. McDougal Littlel/University of Montana.
- The Geometer's Sketchpad Resource Center. KCP Technologies. http://www.dynamicgeometry.com/.