Syllabus - Math 353: Survey of Geometry

Department of Mathematics
Millersville University

Description

Various examples of axiom systems, axiomatic development of neutral geometry followed by Euclidean and hyperbolic geometry. Models for Euclidean and hyperbolic Geometry. Emphasis on proving geometric theorems, both orally and in writing. (3 credits)

Prerequisites

A C- or better in both Math 310 and Math 322 or permission of instructor.

Objectives

The student will:

- Appreciate the historical significance of Euclid's axiomatic treatment of geometry.
- Demonstrate an understanding of axiomatic systems through various examples.
- Have a clear understanding of the set of axioms in neutral, Euclidean, and hyperbolic geometry.
- Understand the models for Euclidean and hyperbolic geometry.
- Write proofs in the context of neutral, Euclidean, and hyperbolic geometry.

Assessment

Students will demonstrate their understanding through work in class, homework, and examinations.

Course Outline

I. Euclid's Elements

The historical significance of Euclid's Elements
A look at Book I of Elements
A critique of Euclid's Elements

II. Axiomatic Systems and Incidence Geometry

The structure of an axiomatic system
An example: incidence geometry
The parallel postulates in incidence geometry
Some theorems from incidence geometry

III. Axioms of Plane Geometry

The undefined terms and two fundamental axioms
Distance and the Ruler Postulate
The Plane Separation Postulate
Angle measure and the Protractor Postulate
The Crossbar Theorem and the Linear Pair Theorem
The Side-Angle-Side Postulate
The Parallel Postulates and models

IV. Neutral Geometry

The Exterior Angle Theorem and existence of perpendiculars
Triangle congruence conditions
Three inequalities for triangles
The Alternate Interior Angles Theorem
The Saccheri-Legendre Theorem
Quadrilaterals
Statements equivalent to the Euclidean Parallel Postulate
Rectangles and defect
The Universal Hyperbolic Theorem

V. Euclidean Geometry

Basic theorems in Euclidean geometry
The Parallel Projection Theorem
Similar triangles
The Pythagorean Theorem

VI. Hyperbolic Geometry

Basic theorems in hyperbolic geometry
Common perpendiculars
The angle of parallelism
Limiting parallel rays
The classification of parallels

VII. Circles

Circles and lines in neutral geometry
Circles and triangles in neural geometry
Circles in Euclidean geometry

VIII. Models

The Cartesian model for Euclidean geometry
The Poincare disk model of hyperbolic geometry

Textbooks: Gerald A. Venema, *Foundations of Geometry* (2nd edition). Pearson Education 2012.

Last Revised: February, 2016