Partial Differential Equations
MATH 467.01 (3 credits)

Prerequisites: A grade of C- or better in MATH 365 (*Ordinary Differential Equations*) is the prerequisite for this course.

Textbook: *A First Course in Partial Differential Equations*, J. Robert Buchanan and Zhoude Shao, World Scientific Publishing Company, Hackensack, NJ USA (late 2017).

Objectives: MATH 467 provides an introduction to partial differential equations and their applications. Upon completion of this course the student will:

- understand how partial differential equations arise in the mathematical description of heat flow and vibration,
- demonstrate the ability to solve initial boundary value problems,
- express and explain the physical interpretations of common forms of PDEs,
- understand issues related to existence and uniqueness of solutions,
- depict in series and graphical form the solutions to initial boundary value problems,
- appreciate the theory underlying the solution techniques,
- be acquainted with applications of partial differential equations in various disciplines of study.

Course Contents: Topics covered in this course may include the following. The material will be presented in a logical order, though not necessarily in the order shown below. Other topics will be added as time and interests allow.

- Introduction
  - **Extremely** brief review of topics from ordinary differential equations
  - Heat equation as model of heat conduction in a rod
  - Separation of variables
  - Fundamental solutions and superposition of solutions
- Fourier series
  - Orthogonality and Euler-Fourier formulas
  - Periodicity
  - The Fourier Convergence Theorem
  - Even and odd functions; sine and cosine series
  - Extensions of functions to even and odd functions
- The Heat Equation
  - Solution of initial/boundary value problems
  - Homogeneous Dirichlet boundary conditions
  - Nonhomogeneous boundary conditions and steady-state solutions
  - Other boundary conditions
  - A Maximum Principle and uniqueness of solution for the heat equation
- The Wave Equation
  - Solution of initial/boundary value problems
  - Characteristic coordinates and a general solution
  - D'Alembert's solution of the initial value problem
  - Energy integrals and uniqueness of solution for the wave equation

- Laplace's Equation
  - Boundary value problems in rectangular coordinates
  - Boundary value problems in polar coordinates
    - Periodic boundary conditions
  - Neumann problems and mixed boundary conditions
    - Lack of uniqueness of solution
    - Necessary conditions for the existence of a solution
  - Uniqueness of solutions
    - Mean Value Property
    - Weak form of the Maximum Principle
    - Uniqueness of solutions of the Dirichlet problem
- Sturm-Liouville Theory
  - General two-point boundary value problem
  - Eigenvalues and eigenfunctions
  - Lagrange's identity and consequences
  - Normalization of eigenfunctions and general eigenfunction expansions
  - Nonhomogeneous boundary value problems

Other topics may be included if time permits.