I. MATH 471 — Mathematical Modeling (3 credits)

II. Catalog Description

Applications of mathematics to real–world problems drawn from industry, research laboratories, the physical sciences, and engineering, and the scientific literature. Techniques used in this course may include parameter estimation, curve fitting, calculus, elementary probability, optimization, computer programming, and ordinary and partial differential equations.

PREREQUISITE: Grade of C– or better in MATH 365 — *Ordinary Differential Equations*

III. Objectives

- To introduce mathematics students to real–world problems.
- To introduce students to problems in disciplines outside of mathematics in which "example", "existence", and "realism" may be more important than "calculation" or "proof".
- To introduce students to the use of the computer as a tool for exploration, simulation, visualization, and presentation of problem solutions.
- To impress upon students the need to acquire the vocabulary and methods of expression found in disciplines outside mathematics, in order to communicate mathematical solutions and treatments of problems to non–mathematicians.
- To give students the opportunity to use skills learned throughout their mathematics and science coursework.

IV. Outline

The content of the course can be viewed either as a set of applied mathematics case studies and mathematical modeling problems and projects or as a set of underlying mathematical modeling techniques which are applicable to a wide variety of real–world problems. Changes in technology, the business climate, and research initiatives of industry and academia may provoke changes in the mathematical modeling topics studied in this course. The topics listed below are merely examples of the topics which may be covered in this course. If a greater need for, expertise in, or applicability of other topics can be found, instructors should feel free to include other topics. The list of topics below does not imply that all of these topics should be covered. The instructor's own background, expertise, and interests coupled with the students' background and interests should guide the selection of topics presented and the amount of detail and sophistication placed in the modeling effort.

A. Mathematical modeling topics
   1. Allometry, shape, and form
      a. Organ sizes and body weights of animals
      b. Geometry of blood vessels
      c. Design of blades in cutting tools
   2. Pharmacokinetic modeling
      a. One compartment linear models
      b. Two compartment linear models
      c. Non–linear Michaelis-Menten processes
      d. Physiologically–based modeling
   3. Chemical reaction dynamics
   4. Population biology and mathematical ecology
      a. Cleaning pollution from a lake
      b. Mutual competition between two species
      c. Predator–prey interactions

    d. Pioneer–climax interactions
5. Traffic dynamics
    a. Paths of turning and towed vehicles
    b. Braking to avoid collisions
    c. Driving in tunnels
    d. Shocks and rarefactions in traffic flow
6. Optimal control and utility
    a. Energy allocation in plants
    b. Energy consumption during animal migration
    c. Employment levels in a company
    d. Harvesting and stocking strategies
    e. Temperature control in reptiles
7. Elections and voting
8. Games and chance and skill
9. Birth and death
    a. Branching processes
    b. Inventory control
    c. Traffic junctions and turn lanes
    d. Justification for pedestrian crossings

B. Mathematical modeling techniques
1. Axiomatic systems
2. Geometry and trigonometry
3. One, two, and three–dimensional ordinary differential equations
4. Numerical solution of ordinary differential equations
5. Parameter estimation and curve fitting
6. Graph theory
7. Planar dynamical systems
    a. Isoclines
    b. Equilibria
    c. ω–limit sets
    d. Vector fields
8. One–dimensional partial differential equations and fluid flow
9. Conservation laws
10. Control theory
11. Elementary probability
12. Development of computer–based simulations

Each of these topics is rich enough to justify at least one complete mathematics course. This course will not attempt to cover all the detail necessary for a deep understanding of these topics, but instead will introduce the key ideas from these topics necessary to build and analyze simple mathematical models. Depending on the backgrounds and academic and career interests of the students and the instructor, topics other than those listed here may be included in this course. One component of the course will be the use of calculators, computers and mathematical software such as *Mathematica* for the creation of simulations, development of visualizations of model behavior, and numerical solution of equations.

At the end of this course, successful students will be able to develop mathematical models to describe various real–world problems and will be better able to tap the expertise and knowledge base of people in other disciplines when a mathematical modeling effort is required in an unfamiliar area. The students will have some experience creating written reports describing their models in language suitable for consumption by non–mathematicians. They will also have experience developing and giving oral presentations geared for a mixed audience of mathematicians and non–mathematicians.

V. Criteria for Evaluating Student Performance

Student grades will be based on class participation, homework exercises, and a final project. Homework may take the form of extending models developed in class, carrying out calculations using an existing model, developing models in contexts not yet presented in class, searching the literature or the internet for background material or previous work on mathematical models in specific application areas, or critiquing existing models. Since in most modern industrial, scientific, and business settings people seldom work in isolation, students will be allowed to work in small groups (perhaps 3 or 4 students per group) on some homework assignments and submit their assignments as a group. However, the final modeling project must a student's individual work. Since one of the objectives of the course is to emphasize the translation and presentation of mathematical ideas in formats appropriate for the non–mathematician, assignments will be formatted using word processing software in the mathematics department's computer laboratory. Students will also be required to present their work orally with appropriate visual aids (e.g. charts, handouts, etc.) in front of the class.

A student's final grade will be based on 1/3 class participation and oral presentation of results, 1/3 written homework, and 1/3 final project.

VI. Texts and Reading Material

Mathematics for Dynamic Modeling, 2nd edition, Edward J. Beltrami, Academic Press, 1998.

Since the list of topics to be covered is broad and textbook publishers offer only a few books suitable for undergraduate courses in mathematical modeling, the textbook used for this course will most likely be heavily supplemented. Journal articles and other printed source material will be made available to the class for reading. Reference and supplementary materials will be placed on reserve in the Ganser Library for use by the students.

VII. General Education Credit

This course may not be taken for general education credit.

References

[1] Bender, E.A., (1978) *An Introduction to Mathematical Modeling*, New York, John Wiley and Sons.

[2] Boyce, W.E. (ed.), (1981) *Case Studies in Mathematical Modeling*, Boston, Pitman.

[3] Friedman, A. and W. Littman, (1994) *Industrial Mathematics: A Course in Solving Real World Problems*, Philadelphia, SIAM.

[4] Giordano, F.R. and Weir, M.D., (1985) *A First Course in Mathematical Modeling*, Monterey, Brooks/Cole.

[5] Harris, T.E., (1989) *The Theory of Branching Processes*, New York, Dover.

[6] James, D.J.G. and McDonald, J.J. (eds.), (1981) *Case Studies in Mathematical Modelling*, New York, John Wiley and Sons.

[7] Klamkin, M.S. (ed.), (1987) *Mathematical Modelling: Classroom Notes in Applied Mathematics*, Philadelphia, SIAM.

[8] Klamkin, M.S. (ed.), (1990) *Problems in Applied Mathematics: Selections from the SIAM Review*, Philadelphia, SIAM.

[9] LeVeque, R.J., (1992) *Numerical Methods for Conservation Laws*, Basel, Birkhäuser Verlag.

[10] Mesterton–Gibbons, M., (1995) *A Concrete Approach to Mathematical Modelling*, New York, John Wiley & Sons.

[11] Sadovskiĭ, L.E. and Sadovskiĭ, A.L., (1994) *Mathematics and Sports*, Providence, RI, American Mathematical Society.

[12] Strang, G., (1986) *Introduction to Applied Mathematics*, Wellesley, Wellesley–Cambridge.

[13] Wolfram, S.W., (1991) *Mathematica: A System for Doing Mathematics by Computer*, New York, Addison–Wesley.