

## I. MATH 472 — Financial Mathematics (3 credits)

## II. Catalog Description

The mathematical analysis of investment emphasizing the time value of money, rates of return for investment cash-flow sequences, utility functions, stochastic processes, mean–variance analysis, portfolio selection, hedging strategies, the capital assets pricing model, and the Black-Scholes theory of options. This course will also introduce some of the topics covered on actuarial exams SOA/FM and SOA/MFE administered by the Society of Actuaries.

PREREQUISITE: Grade of C– or better in MATH 311 — Calculus III<sup>1</sup>.

## III. Objectives

The objectives of this course include introducing the students to the mathematical treatment of risk-neutral valuation, arbitrage, options, stochastic modeling, and derivatives. Upon successful completion of this course students will be able to:

- price popular types of financial derivatives such as Put and Call options and create financial strategies which use these derivatives to reduce risk,
- mathematically model phenomena using stochastic processes and geometric Brownian motion,
- approximate discrete stochastic processes by continuous stochastic processes and *vice versa*,
- apply the “efficient market hypothesis” and the Arbitrage Theorem and their implications in various financial modeling situations,
- solve the Black-Scholes equation with the appropriate boundary and final/initial conditions to model the value of a financial derivative or combination of derivatives.

Much of this theory coalesced in the 1970’s in work done by Black, Scholes, and Merton and which earned them the Nobel Prize for Economics in 1997. Students will appreciate how a clever, yet straight forward, application of undergraduate mathematics can yield such profound theoretical results and enable the creation of the world’s derivative markets. Upon successful completion of this course interested students should find themselves with sufficient background to pursue more advanced study in mathematical finance in graduate schools. Students will be introduced to some of the topics on actuarial examinations, SOA/FM and SOA/MFE.

## IV. Outline

The semester activities may include exposure to and exploration of the following topics. Instructors should feel free to add other topics as interest dictates.

Partial Topic List:

- Review of interest rates and present value analysis: rates of return and continuously varying interest rates.<sup>2</sup>

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<sup>1</sup>Some background in elementary statistics is recommended. A brief overview of relevant topics from probability and statistics is included in the course.

<sup>2</sup>Includes material covered in the Actuarial Examination SOA/FM.

- General principles: derivatives and options, utility functions, risk aversion, linear pricing, portfolio choice, finite models, risk-neutral pricing.
- Elementary probability: probabilities and events, conditional probability, random variables, expected value, variance, covariance, and correlation.
- Normal random variables: continuous random variables, properties of normal random variables, the Central Limit Theorem.
- Lognormal random variables: properties of lognormal random variables, their use as a model for changes in the value of a security.
- Brownian motion: geometric Brownian motion and its development as a limit of simpler models, Itô's Lemma.<sup>3</sup>
- Fixed-income securities: value formulas, bond details, yield, duration.
- Term structure of interest rates: yield curve, forward rates, floating rate bonds.<sup>1</sup>
- Capital asset pricing model: market equilibrium, capital market line, security market line, investment implications.<sup>1</sup>
- Examples of pricing contracts via arbitrage.
- The Arbitrage Theorem: the Dual Theorem of Linear Programming, the Fundamental Theorem of Financial Mathematics and multiperiod binomial models.
- The Black-Scholes formula: the Black-Scholes partial differential equation and its boundary and initial/final conditions, properties of Black-Scholes option cost, estimating the volatility parameter, and pricing Put Options.<sup>1</sup>
- Valuing by expected utility: limitations of arbitrage pricing, portfolio selections, estimating covariances, mean variance analysis of risk-neutral-priced Call Options, and single period rates of return.<sup>1</sup>
- Interest rate derivatives: examples, theory, and pricing applications,

## V. Criteria for Evaluating Student Performance

Student grades may be based on class participation, homework exercises, tests, and a final examination. Homework may be taken from the textbook (if one is used) and may be supplemented by exercises from other texts or exercises created by the instructor. Some portion of the homework should consist of extended assignments involving the collection of historical data on financial instruments such as stock, bond, option, future, or derivative prices and the analysis of that data for the formulation of investment, leveraging, and hedging strategies. The following table lists suggestions for evaluation instruments associated with the course objectives described above.

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<sup>3</sup>Includes material covered in the Actuarial Examination SOA/MFE.

<b>Objective</b>	<b>Evaluation Instrument</b>
Use of discrete processes in financial modeling	Collection of data from the world wide web, computer-based homework
Use of continuous processes in financial modeling	Homework, tests, final examination
Knowledge of the relationship between stochastic and deterministic models	Homework, tests, final examination
Understanding the roles of Put and Call options in risk reduction	Homework, tests, final examination
Understanding of hedging strategies to reduce risk	Homework, tests, final examination
Understanding the role of arbitrage and its absence in investments	Homework, tests, final examination
Understanding the role of the Black-Scholes partial differential equation and its boundary and final conditions in option pricing	Homework, tests, final examination

## VI. Reading Material

Textbooks on financial modeling written at the appropriate level for typical undergraduates are in short supply. Many assume knowledge of measure theory and stochastic calculus despite their claims of having been written for an undergraduate audience. However, several elementary books exist, which develop the topics of this proposed course from knowledge of multivariable calculus and elementary probability and statistics. Since this proposed course has no probability and statistics prerequisite, the instructor may have to supplement textbooks which omit introductory material on those topics. Suggested textbooks for this course include:

- *An Introduction to Mathematical Finance: Options and Other Topics*, Sheldon M. Ross, Cambridge University Press, Cambridge, 1999.
- *Investment Science*, David G. Luenberger, Oxford University Press, New York, 1998.
- *The Mathematics Of Financial Derivatives: A Student Introduction*, Paul Wilmott, Sam Howison, and Jeff Dewynne, Cambridge University Press, Cambridge, 1995.
- *An Undergraduate Introduction to Financial Mathematics*, 3rd edition, J. Robert Buchanan, World Scientific Publishing Company, Hackensack, NJ USA 2012.

These books may be supplemented by instructor-designed handouts and web-based resources.

## VII. General Education Credit

This course may not be taken for general education credit.

## References

- [1] Marco Avellaneda, ed., *Quantitative Analysis in Financial Markets*, World Scientific Publishing, River Edge, New Jersey, 1999.

- [2] Martin Baxter and Andrew Rennie, *Financial Calculus: An Introduction to Derivative Pricing*, Cambridge University Press, Cambridge, 1996.
- [3] J.W.S. Cassels, *Economics for Mathematicians*, London Mathematical Society Lecture Note Series, Number 62, Cambridge University Press, Cambridge, 1981.
- [4] Morton D. Davis, *The Math of Money: Making Mathematical Sense of Finance*, Springer-Verlag, New York, 2001.
- [5] Robert J. Elliott and P. Ekkehard Kopp, *Mathematics of Financial Markets*, Springer-Verlag, New York, 1999.
- [6] John C. Hull, *Options, Futures, and Other Derivatives*, 4th edition, Prentice-Hall, Inc., Upper Saddle River, NJ, 2000.
- [7] John C. Hull, *Fundamental of Futures and Options Markets*, 4th edition, Prentice-Hall, Inc., Upper Saddle River, NJ, 2002.
- [8] Roger Lowenstein, *When Genius Failed: the Rise and Fall of Long-Term Capital Management*, Random House, Inc., New York, 2000.
- [9] David G. Luenberger, *Investment Science*, Oxford University Press, New York, 1998.
- [10] Thomas Mikosch, *Elementary Stochastic Calculus With Finance in View*, World Scientific Publishing, River Edge, New Jersey, 1998.
- [11] Salih N. Neftci, *An Introduction to the Mathematics of Financial Derivatives*, Academic Press, San Diego, 1996.
- [12] Eliezer Z. Prisman, *Pricing Derivative Securities*, Academic Press, San Diego, CA, 2000.
- [13] Sheldon M. Ross, *An Introduction to Mathematical Finance: Options and Other Topics*, Cambridge University Press, Cambridge, 1999.
- [14] Steven S. Skiena, *Calculated Bets: Computers, Gambling, and Mathematical Modeling to Win*, Cambridge University Press, Cambridge, 2001.
- [15] J. Michael Steele, *Stochastic Calculus and Financial Applications*, Springer-Verlag, New York, 2001.
- [16] Johannes Voit, *The Statistical Mechanics of Financial Markets*, Springer-Verlag, New York, 2001.
- [17] Paul Wilmott, Sam Howison, and Jeff Dewynne, *The Mathematics of Financial Derivatives: A Student Introduction*, Cambridge University Press, Cambridge, 1995.